A New and Robust Subgame Perfect Equilibrium in a model of Triadic Power Relations

by

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Abstract:
We present a new subgame perfect equilibrium in an infinitely repeated game, which has Basu's (1986) triadic model as the stage game. The payoff for the laborer is the same as in Basu's model. The equilibrium is more robust than the solution in Naqvi and Wemhöner (1995) in the sense that the equilibrium does not require the same high degree of rationality; simple well-known strategies are applied, and both the landlord and the merchant are better off than in the stage game. In the equilibrium outcome the merchant receives a share of the extra profit from the extortionary labor contract.

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1. Introduction

Basu (1986) identifies what he describes as an "equilibrium" in a stage game of three players where a laborer accepts a wage below his ordinary reservation wage (the game will be presented below). Naqvi and Wemhöner (1995) demonstrate that, while this "equilibrium" is a Nash equilibrium, it is not subgame perfect. They go on to identify a subgame perfect Nash equilibrium of the game. In that equilibrium, it turns out that the laborer receives his ordinary reservation wage.

Naqvi and Wemhöner actually proceed by developing an alternative model where the stage game is repeated infinitely, and propose a strategy profile which has Basu's outcome as a subgame perfect Nash equilibrium. The present paper is a comment on the robustness of their equilibrium, and it proposes an alternative solution, which is in sharp contrast to Naqvi and Wemhöner's equilibrium. Their equilibrium requires a complex strategy profile and a rather extreme form of rationality, where the landlord has to convince the merchant that the laborer will not play his stage game equilibrium strategy, but will rather contribute in maintaining the worst outcome for himself.

If we would like to identify a more robust equilibrium strategy profile which would result in an outcome in line with Basu's (1986) Nash equilibrium¹, we have to look for (1) an alternative strategy profile for the infinitely repeated game, or (2) an alternative specification of the stage game. Independently of us, Basu (2000) has solved the problem applying the

¹ It is our impression that the empirical support for this equilibrium is strong for poor rural economies. Basu (1997 and 2000) discusses alternative applications of the triadic model.
second approach\textsuperscript{2}. In this paper we apply the first approach, that is, we identify an alternative strategy profile for the repeated game, which constitutes a subgame perfect equilibrium. The outcome for the laborer is the same as in Basu (1986), while the merchant is better off, and consequently the landlord is worse off. The equilibrium is robust in the sense that simple well-known strategies are applied, the landlord can punish the merchant directly without involving the laborer, and finally both the merchant and the landlord are better off than in the stage game. The game that we consider is identical to that of Naqvi and Wemhöner (1995), except that we allow the landlord to give the merchant a reward.

2. The model

The stage game has the following timing and available actions. 1) The landlord offers a wage \( w_i \). 2) The laborer accepts (a) or rejects (r) the offer. 3) The merchant trades (t) or does not trade (n) with the laborer. 4) The landlord trades or does not trade (N) with the merchant, and if he trades, he will trade at the reservation terms (T) or at more favorable terms (F) for the merchant. Figure 1 illustrates the game for the case where the landlord offers \( w_0 \). We only include payoffs for the equilibrium outcomes discussed in this paper. The figure is similar to Figure 1 in Naqvi and Wemhöner (1995). However, we have an additional action F available for the landlord at the last decision node\textsuperscript{3}. Also note that the notation differs when it comes to the merchant's decision.

\textit{Figure 1, about here.}

\textsuperscript{2} The line taken by him is very different from the one adopted in this paper.

\textsuperscript{3}
Strategies can be conditional on previous actions in the stage game, and we write sequences of actions as in the example (w_i, r, n, T) denoting that the laborer rejects w_i, the merchant does not trade with the laborer, and the landlord trades with the merchant. Payoffs for the landlord, merchant and laborer are functions of the equilibrium outcome.

Let us define the worker's utility function to be the mapping, \( U: \{r\} \cup \mathbb{R}_+ \times \{n, t\} \to \mathbb{R} \). Thus \( U(r, n) \) is the utility that the laborer gets when he rejects the landlord's offer (an action which is symbolized by \( r \)) and the merchant refuses to trade with him. He gets the utility \( U(w_i, n) \) when he accepts a wage offer \( w_i \) and the merchant refuses to trade with him. Similarly, the laborer gets the utility \( U(r, t) \) or \( U(w_i, t) \) when the merchant agrees to trade with him. We define the wage \( w_1 \) characterized by \( U(r, t) = U(w_1, t) \), as the laborer's ordinary reservation wage. Basu defines the wage \( w_0 \) characterized by \( U(r, n) = U(w_0, t) \), which we term the extortionary reservation wage. We redefine \( w_0 \) such that \( U(w_0, t) \) is infinitesimally better than \( U(r, n) \). This is to avoid any discussions of robustness, which would be due to an indifferent laborer. Since the difference is infinitesimal, we apply \( w_0 \) as an approximation to the extortionary wage in Basu (1986) and Naqvi and Wemhöner (1995). Furthermore, we make the natural assumption that the laborer prefers to trade with the merchant, which implies \( w_1 > w_0 \).

In the Nash equilibrium identified by Basu (1986) the landlord offers \( w_0 \) and plays \( N \) after the sequence \( (r, t) \) and \( T \) otherwise. The merchant plays \( n \) after \( r \), and \( t \) otherwise. The laborer plays \( r \) after \( w_i < w_0 \), and \( a \) otherwise. The equilibrium outcome is the sequence \( (w_0, a, t, T) \).

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3 We need all three actions to be able to present both the strategy profile applied by Naqvi and Wemhöner and the strategy profile applied by us. To present our strategy profile only, we do not need action \( N \).
Using a notation in line with the laborer's utility function, the respective payoffs for the landlord, the merchant and the laborer becomes,

\[ V(w_0, T), W(T, t) \text{ and } U(w_0, t). \]

It is easy to check that no player has an incentive to deviate. However, as noted by Naqvi and Wemhöner (1995), this is not a subgame perfect equilibrium. Naqvi and Wemhöner identified the only subgame perfect equilibrium of the stage game, where \textit{the landlord offers } \( w_1 \text{ and plays } T, \text{ the merchant plays } t \text{ and the laborer plays } r \text{ after } w_i < w_1, \text{ and } a \text{ otherwise.} \) \ We will use the notation \( E_0 \) for this strategy set. The equilibrium outcome is the sequence \((w_1, a, t, T)\), which leads to the payoffs,

\[ V(w_1, T), W(T, t), U(w_1, t). \]

Furthermore, Naqvi and Wemhöner (1995) argue that the Basu (1986) equilibrium can be sustained as a subgame perfect equilibrium in an infinitely repeated game. Below we formulate the equilibrium strategies from inequalities (18) and (19) in their paper\(^4\). We specify the strategies in terms of an initial equilibrium path and player specific punishment paths, in line with Abreu (1988). We will use the notation \( E_1 \) for this strategy set.

**Naqvi and Wemhöner's subgame perfect strategy profile.**

\(^4\) Using our notation (18) becomes \( W(T, t) \geq (1-\Delta)W(N, t) + \Delta W(T, t) \), where \( \Delta = \delta^i \), and \( \delta \) is the merchant's discount factor, and (19) becomes \( V(w_1, T) \leq (1-\Delta_i)V(r, N) + \Delta_i V(w_0, T) \), where \( \Delta_i = \delta^d_i \), and \( \delta_i \) is the landlord's discount factor.
Suppose the merchant is sufficiently myopic and the landlord is sufficiently patient, as specified by inequalities (18) and (19) in Naqvi and Wemhöner (1995), then the following strategy profile constitutes a subgame perfect Nash equilibrium:

Equilibrium path:
- Play according to the sequence \((w_0, a, t, T)\).

The laborer specific punishment path (which applies whenever the laborer plays \(r\) after \(w_0\)):
- Play the sequence \((n, T)\).

The merchant specific punishment path (which applies whenever the merchant plays \(t\) after \((w_0, r)\)):
- Play according to the sequence \((w_0, r, t, N)\) for \(d\) periods and then return to the equilibrium outcome \((w_0, a, t, T)\).

The landlord specific punishment path (which applies whenever the landlord plays \(T\) after \((w_0, r, t)\)):
- Play according to the stage game outcome \((w_1, a, t, T)\).

Consequently Naqvi and Wemhöner (1995) presented two subgame perfect equilibria, the one presented above leading to the equilibrium outcome \((w_0, a, t, T)\), which is the same as in the Nash equilibrium identified by Basu (1986), and the subgame perfect equilibrium of the stage game, having the equilibrium outcome \((w_1, a, t, T)\).

In Proposition 1 we present a third subgame perfect equilibrium of the repeated game. We apply the discount factor \(\delta\) for the merchant. The sufficient conditions in Proposition 1 are in
Corollary 1 specified for monetary payoffs. We will use the notation $E_2$ for the strategy set applied in Proposition 1.

**Proposition 1.**

Suppose $F$ satisfies $V(w_0, F) > V(w_1, T)$ and $(1 - \delta)W(T, n) + \delta W(F, t) > W(T, t)$, then the following strategy profile constitutes a subgame perfect Nash equilibrium:

**Equilibrium path:**

- Play according to the sequence $(w_0, a, t, F)$.

The laborer specific punishment path (which applies whenever the laborer plays $r$ after $w_0$):

- Play the sequence $(n, T)$ and then return to the equilibrium outcome $(w_0, a, t, F)$.

The merchant and landlord specific punishment path (which starts whenever one of them deviates from the equilibrium outcome or the laborer's punishment path):

- Play according to the stage game outcome $(w_1, a, t, T)$.

**Proof:** The laborer will not deviate from the equilibrium outcome as long as $U(w_0, t)$ is infinitesimally better than $U(r, n)$. The merchant will not deviate from the laborer's punishment path as long as $(1 - \delta)W(T, n) + \delta W(F, t) > W(T, t)$. A necessary condition for this to be true is for $F$ to be a superior action to $T$ from the merchant's point of view, which in turn implies that the merchant will not deviate from the equilibrium outcome. The landlord will not deviate from the equilibrium outcome as long as $V(w_0, F) > V(w_1, T)$, and not from the labor specific punishment, since $T$ is his myopic best response. By definition, no player will deviate from the subgame perfect stage game equilibrium. ||
Note that the outcome for the laborer is the same as in Basu (1986). However, the payoffs for the merchant and the landlord from F will be significantly different from their respective payoffs from T. To illustrate this, we study the realistic case where the payoff functions are monetary and consequently separable, i.e. we have \( W(F, t) = F + t, \ W(T, t) = T + t, \) and \( W(T, n) = T, \) for the merchant, and \( V(w_0, F) - V(w_1, T) = (w_1 - w_0) - (F - T) \) for the landlord. Here, F is the monetary surplus for the merchant from trading with the landlord at favorable terms, and T is the monetary surplus from trading at market terms. Consequently the difference \( F - T \) is the loss for the landlord from the favorable terms. Furthermore, t is the monetary surplus for the merchant from trading with the laborer. Now we can formulate Corollary 1.

**Corollary 1.**

*Suppose monetary payoff functions are applied in the sufficient conditions for a subgame perfect equilibrium in Proposition 1, then these conditions become,*

\[
(w_1 - w_0) > (F - T) > t (1 - \delta)/\delta.
\]

**Proof:** We have \( V(w_0, F) - V(w_1, T) = (w_1 - w_0) - (F - T) > 0, \) and \( (1 - \delta)W(T, n) + \delta W(F, t) - W(T, t) = (1 - \delta)T + \delta (F + t) - T - t = \delta (F - T + t) - t > 0. \) Combining the two conditions we have \( (w_1 - w_0) > (F - T) > t (1 - \delta)/\delta. \)

Corollary 1 leads to Corollary 2.
Corollary 2.

Suppose the sufficient conditions from Corollary 1 are satisfied, and suppose the landlord is maximizing surplus, then we necessarily have

\( a) (w_1 - w_0) > t (1 - \delta)/\delta \)

\( b) F - T = t (1 - \delta)/\delta \)

Proof: a) is a direct implication of Corollary 1. b) is a direct implication of Corollary 1, when the landlord maximizes surplus, i.e. minimizes F. ||

So, if the merchant is sufficiently patient, that is Corollary 2a is true, then the landlord will be able to select the F identified by Corollary 2b, such that the equilibrium outcome identified in Proposition 1 will be subgame perfect. Note that the more patient is the merchant, the smaller is the necessary reward, \( F - T = t (1 - \delta)/\delta \), where \((1 - \delta)/\delta\) is the merchant's discount rate.

Corollary 2 makes it transparent that we apply trigger strategies in line with Friedman (1971), for the landlord-merchant game. Both players are better off than in the stage game equilibrium, and for a certain discount factor for the merchant, his minimum reward can be specified as in Corollary 2b.

We have identified three alternative subgame perfect equilibria of the infinitely repeated triadic game, i.e. the equilibrium of the stage game (repeated infinitely), which results from the strategy set \( E_0 \) and leads to the outcome defined by the sequence \((w_1, a, t, T)\), the equilibrium identified by Naqvi and Wemhöner, which results from the strategy set \( E_1 \) and
leads to the outcome \((w_0, a, t, T)\), and the equilibrium identified by us, which results from the strategy set \(E_2\) and leads to the outcome \((w_0, a, t, F)\).

In the remaining part of the paper, we discuss which of the three equilibria is most likely to be selected. Remark 1 comments on the fact that a landlord-merchant coalition would prefer \(E_2\) to \(E_0\) and consequently would profit from a joint deviation from \(E_0\), using their \(E_2\) strategies. Remark 2 comments on the complexity and very high degree of rationality required in \(E_1\).

First note that both the merchant and the landlord will be better off by using \(E_2\) instead of \(E_0\). Since there are only three players, the laborer will also comply with \(E_2\) if the merchant-landlord coalition apply \(E_2\). Furthermore, the timing is such that there is no doubt when the landlord actually deviates from \(E_0\). The landlord-merchant coalition may deviate from \(E_0\) in the following way. The landlord announces that he will use his \(E_2\) strategy, and then he demonstrate this by playing the sequence \((F, w_0)\). Next, the merchant announces that he will also comply with \(E_2\). Since the pair of strategies implies a credible threat, the laborer complies, and \(E_2\) is enforced. The equilibrium strategy profile \(E_1\), suggested by Naqvi and Wemhöner, does not have this property. The merchant will for a certain history even be worse off by using \(E_1\) instead of \(E_0\). Now we can formulate remark 1.

**Remark 1**

*The merchant-landlord coalition is likely to deviate from \(E_0\) by using their \(E_2\) strategies, and thus enforce the \(E_2\) outcome. The coalition is not likely to deviate from \(E_0\) by using \(E_1\).*
Next, consider the history \((w_0, r, t)\), and a landlord who considers not to punish the merchant. If he does not punish, his strategy is not credible and he will consequently infinitely have the stage game payoff \(V(w_1, T)\). If he does punish according to \(E_2\) or \(E_1\), he will have the respective payoffs:

\[\]

\[^{5}\text{Note the relation to Aumann’s (1959) notion of a strong equilibrium, see Fudenberg and Tirole (1991), p. 22: An equilibrium is not strong if a subset of players, taking the actions of others as given, has incentive to jointly deviate.}\]
E₂: \( V(w_1, T) \) infinitely.

E₁: \( V(r, N) \) for \( d \) periods and \( V(w_0, T) \) thereafter.

According to E₂ the landlord will punish the merchant by playing \( (w_1, T) \) infinitely, leading to the subgame perfect stage game equilibrium outcome. According to E₁ the landlord will play N for \( d \) periods. During all these periods the laborer will reject the wage offer, and consequently the landlord will have his minimum payoff \( V(r, N) \). This punishment path is profitable for the landlord, only because the laborer will suddenly accept the wage offer after \( d \) periods. Now we can formulate remark 2.

**Remark 2.**

*Suppose the landlord considers to punish the merchant after the history \((w_0, r, t)\). Then E₂ requires the simple and robust stage game equilibrium strategy, while E₁ requires every player to believe that the laborer will accept \( w_0 \) after \( d \) periods, although he has recently rejected \( w_0 \).*

The remark is on the complexity and the extreme form of rationality assumed by Naqvi and Wemhöner (1995) in E₁. For the punishment to be profitable for the landlord, he has to believe that the laborer will accept \( w_0 \) after the \( d \) periods of punishment, even though the laborer has just rejected \( w_0 \). Furthermore the landlord knows that the laborer will only accept \( w_0 \) if he believes that the landlord believes that he will accept, and so on in an infinite regress. This is an extreme example of history insensitivity in extensive games, as discussed e.g. by Basu (1988). The equilibrium in Proposition 1 is on the other hand not vulnerable to this
critique since the payoff for the landlord from punishing the merchant is independent of the laborer's actions.

A similar critique may be raised against the merchant's punishment of the laborer in Proposition 1, i.e. the punishment is only profitable if the merchant believes that the landlord will return to the equilibrium outcome after the punishment. However, this critique is not relevant since the landlord has not deviated from his equilibrium strategy, and since he has no myopic incentive to deviate.

Remarks 1 and 2 are concerned with equilibrium selection. The first remark demonstrates that the merchant and the landlord have incentives to deviate jointly from the stage game equilibrium, and furthermore it demonstrates that deviation leads to the equilibrium in Proposition 1. In the second remark we argue that the complex equilibrium profile proposed by Naqvi and Wemhöner requires an unlikely high degree of rationality. The equilibrium proposed in Proposition 1, is more likely to exist due to the simple and robust strategy profile.

3. Conclusions

We have proposed a new subgame perfect equilibrium of the repeated version of Basu's (1986) model of triadic power relations. The equilibrium is more robust than the strategy profile proposed by Naqvi and Wemhöner (1995). The equilibrium is also the self-enforcing outcome of a joint deviation by the merchant and the landlord from the stage game equilibrium. We find it satisfying to have a robust equilibrium in line with the realistic (extortionary) labor contract in Basu (1986). The equilibrium we identify differs from Basu (1986) and Naqvi and Wemhöner (1995) when it comes to the distribution of the extra profit
from the extortionary labor contract. In equilibrium the merchant will have a significantly larger payoff, since F is likely to be significantly better for the merchant than T. Intuitively this makes sense, the merchant and the landlord need to collude to be able to extort from the laborer, and in equilibrium they consequently need to share the extra profit to secure a sustainable and robust collusion.
Figure 1. The stage game.
References


