Tacit Collusion in a Segmented and Capacity Constrained Informal Rural Credit Market.

by

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Abstract:
To be able to describe informal rural credit markets, we apply the Brock and Scheinkman (1985) model of a price setting and capacity constrained oligopoly, where firms tacitly collude on monopoly pricing. We generalise the model to allow for third-degree price discrimination. The interval of aggregate lending capacity that supports a counter-intuitively increasing part of the equilibrium price-function is smaller than for the uniform price case. We show that the interval is small even for the uniform price case. Consequently, we cannot expect to identify a significantly increasing part of the price-function in empirical analyses.

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1. Introduction.

Stylized facts indicate that informal rural credit markets in less developed economies are characterised by third-degree price discrimination, which means that the interest rates charged by the informal lenders depend on borrowers' observable characteristics. Basu (1997) demonstrates that dispersion in interest rates can be due to interlinkages with other markets. It can also be due to implicit gift-exchanges. The present paper does not study these kinds of credit interactions, but rather pure credit contracts.

In the case of pure credit contracts, the informal interest rates tend to be very high compared to the commercial bank rate. The gap might be due to hidden costs of lending, which in turn might be due to the risk of default among borrowers that cannot raise collateral, see for example Bose (1998) and Hoff and Stiglitz (1997). However, low default rates indicate small risk-premiums in equilibrium, and informal lenders seem to be well-informed about their borrowers. This can in turn be explained by repeated social and economic interactions within the village. Alternatively, it can be explained by screening efforts, as described in Aleem (1990). The latter explanation may be true for professional moneylenders, but most lenders are likely well informed when their tenants, customers or neighbours ask for loans.

The present paper focuses on other factors that can explain the high interest rates. Basu and Bell (1991) model the markets as fragmented duopolies. The present paper is related to that model in the sense that we study oligopolistic competition in fragmented markets. Compared to Basu and Bells' model, we simplify by assuming no overlap between the local markets. This is to allow for more complex oligopolistic competition within every market. Stylized facts indicate that informal rural markets are characterised by capacity constraints and repeated interactions. We thus find it realistic to apply the Brock and Scheinkman (1985) model of capacity constrained oligopolistic pricing. This model has not been applied to informal rural credit markets. Stylized facts also indicate price discrimination. The main
contribution of the present paper is to generalise the Brock and Scheinkman (1985) model to allow for third-degree price discrimination.

In a separate paper, Hatlebakk (2000), we test the model and also the theories of hidden costs discussed above, using data from a national household survey from Nepal. We find only marginal support for cost-pricing. On the other hand we find strong support for capacity constrained pricing, which implies that lenders have positive profit in equilibrium. For non-constrained villages, the results indicate that lenders tacitly collude on monopoly pricing. We do not find significant support for the increasing part of the price-function in Brock and Scheinkman (1985). As we will see in the present paper, this should be expected.

When we first estimated the uniform price Brock and Scheinkman (1985) model, we had to apply ad-hoc explanations for the observed price-differentials. However, we could not be sure that the empirical specification of the model was true for the multi-segment case. The present paper fills the theoretical gap. The paper demonstrates that the counter-intuitive increasing part of the equilibrium price-function is less likely to be identified in the multi-segment case. The paper even demonstrates that the increasing part is not likely to be identified in the one-segment case.

The stage-game of the Broch and Scheinkman (1985) model is described in Kreps and Scheinkman (1983). When we generalise the stage game to the multi-segment case we apply Ben-Zvi and Helpman (1992). Heterogeneous capacity in the stage game was already incorporated in Kreps and Scheinkman (1983), but was not modelled in the repeated game of Brock and Scheinkman (1985). Allowing for heterogeneous capacities does not add significantly to the theoretical analysis, but describes a prevalent stylized fact for informal rural credit markets. The contribution of the present paper is the generalisation of the repeated game in Broch and Scheinkman (1985) to the multi-segment case.
Section 2 presents the stage game with the equilibrium described in Proposition 1. Section 3 presents the repeated game model for the one-segment case with the equilibrium described in Proposition 2. Except for heterogeneous capacities, the model is the same as the theoretical model reported in Hatlebakk (2000), which in turn is basically the Brock and Scheinkman (1985) model. Section 4 presents the general model of third-degree price discrimination with the equilibrium described in Proposition 3. The one-segment model in section 3 illustrates the general model in section 4. Section 5 concludes.

2. The stage game.

We study an oligopoly with N lenders, having a constant marginal cost c of lending. Total lending capacity is given by $K = \sum_{i=1}^{N} k_i$, where $k_i$ is individual lending capacity, and $k_N$ is the largest lending capacity. Any change in $k_i$ is exogenous and not anticipated by the lenders. This market structure can be due to the fact that informal lenders are borrowers in the formal sector themselves, where they are credit constrained and pay an interest rate c. Any change in $k_i$ is thus due to increased formal lending.

We will now describe the information structure of the model. Informal lenders will only lend to borrowers they know well from repeated social and economic interaction. The information and enforcement structure is such that lenders who operate in the market lend at no risk, while outsiders cannot lend with profit. The repeated interaction implies that borrowers will never ask for larger loans than they are able to repay. Lenders have common knowledge about the household characteristics needed to categorise households into segments. However, lenders do not know households' individual demand curves. This implies that third-degree price discrimination is possible, while perfect price discrimination is not an option. Potentially net-borrowers can profit from arbitrage, by re-lending their loans in higher priced segments.
However as other lenders, they need to be able to enforce the contract. We assume that a net-borrower is only able to enforce contracts within his own segment\(^1\).

The information structure implies that borrowers can only have loans from a restricted number of lenders. As modelled in Basu and Bell (1991) the set of lenders might not be the same for all borrowers. To simplify the present model, we assume that the local markets do not overlap, that is, the N lenders operate in the same market and only in this market\(^2\). The local market is likely to be geographically concentrated, and within a village there may be more than one market of the type modelled in the present paper.

Next we will describe the demand structure. Lenders meet the segment-specific inverse demand curves for loans, \( p_s = a_s - b_s q_s \). We sort segments according to \( a_s \), with the largest intercept being \( a_w \). For any uniform price \( p \leq a_1 \), we assume that the aggregate demand curve is linear, i.e. \( q = a - p = \sum_{s=1}^{w} q_s(p) = \sum_{s=1}^{w} (a_s - p) / b_s \). Inserting for \( p = 0 \), we have \( \sum_{s=1}^{w} a_s / b_s = a \), and \( \sum_{s=1}^{w} 1 / b_s = 1 \). Thus for any uniform \( p \leq a_1 \) the aggregate demand will be the same as in a one-segment model with the demand curve \( q = a - p \). For \( p > a_1 \) the aggregate demand curve is described by partial linear parts where the curve bends at every \( a_s \), that is, when demand becomes zero in segment s. The aggregate demand curve is illustrated for the two-segment case by the solid curve in Figure 1.

\textit{Figure 1 about here.}

\(^1\) Net-lenders will typically have more options available for enforcement than net-borrowers, due to larger wealth and positions as landowners, employers or shop-keepers. Net-borrowers might have some enforcement power due to social interaction, but this is likely restricted to their peers. From Hatlebakk (2000) we know that caste is a main indicator of segments in Nepal.

\(^2\) The equilibrium allocation of loans may still appear fragmented, since lenders will not interact with every borrower, although this is an available option.
In the two-segment case the inverse aggregate demand curve for \( p > a_1 \) equals the inverse demand curve for segment 2. For \( p < a_1 \), there is positive demand from segment 1, and the inverse aggregate demand curve bend at the point A in Figure 1, where \( p = a_1 \).

The one-segment version of our model covers the pure one-segment case where the aggregate inverse demand curve is actually, \( p = a - q \), and the uniform price multi-segment case. In the latter case we will only allow \( p \leq a_1 \). This is to simplify the multi-segment model. A specification of the inverse aggregate demand curve for \( p > a_1 \) would vary with the number of segments having zero demand. When only one segment has positive demand the inverse demand curve is \( p = a_w - b_w q \), when two segments have positive demand it is, \( p = \frac{(a_w b_{w-1} + a_{w-1} b_w)}{(b_w + b_{w-1})} - \frac{(b_w b_{w-1}) q}{(b_w + b_{w-1})} \). We avoid this complex notation, by only allowing \( p \leq a_1 \).

Furthermore we will allow values of \( K \), where the equilibrium price function will be \( p = a - K \). We thus have the necessary assumption \( K \geq K_0 = a - a_1 \). So when we in the propositions characterise the equilibrium prices as a function of \( K \), the results for the multi-segment case are only true for \( K \geq K_0 \), while the results for the one-segment case are true for all \( K \). Although this is the case, the generalisation to \( K < K_0 \) is usually rather obvious, and in illustrating the propositions we will draw the likely price-function even for \( K < K_0 \).

We will now apply Figure 1 to illustrate the equilibrium consumer surplus. With the linear inverse demand curve \( p = a - q \), the optimal uniform monopoly price becomes \( \frac{a + c}{2} \), the optimal monopoly quantity becomes \( \frac{a - c}{2} = K_1 \). The Brock and Scheinkman (1985) model, which is applied to the informal credit market by Hatlebakk (2000), identifies the intervals of \( K \) for which \( \frac{a + c}{2} \) is sustainable by tacit collusion. Brock and Scheinkman demonstrated that for \( K < K_1 \) the equilibrium uniform price equals the capacity determined price, \( p = a - K \), which is also the competitive price. In the present paper we will demonstrate that price discrimination is sustainable even for \( K < K_1 \). We thus need a non-empty interval below \( K_1 \) for the multi-segment case, and we assume \( K_0 < K_1 \). This assumption is equivalent
to $a_1 > p^* = (a + c)/2$. The assumption implies that the segment $s = 1$ will have positive demand at the uniform monopoly price. This is a technical assumption, since we can imagine segments having $a_s$ below $p^*$ that are not included in the model.

Restricting the multi-segment case to $K \geq K_0$, implies a simple notation for consumer surplus, which is illustrated for the two-segment case in Figure 1. In the one-segment case consumer surplus is the area between the inverse demand curve $p = a - q$, and the equilibrium price. In the multi-segment case, there is an additional surplus represented by the area between the one-segment inverse demand curve $p = a - q$, and the solid aggregate inverse demand curve, given by the triangle $(a_2Aa)$. For the general multi-segment case, we denote this triangle as $\Delta/2$, where $\Delta = \sum_{s=1}^{w} \frac{a_s^3}{b_s} - a^2$.

The extra consumer surplus $\Delta/2$ is the main difference between a pure one-segment model and a uniform price multi-segment model. The uniform price will be the same in the two models for $K \geq K_0$, while there is an extra consumer surplus of $\Delta/2$ in the multi-segment model. The multi-segment model reduces to the pure one-segment model if all $a_s = a$.

The extra consumer surplus of $\Delta/2$ is what lenders would like to extract by price discrimination. If lenders are able to sustain optimal price discrimination, we will see below that they will always be able to increase their profit by half this area, that is $\Delta/4$. It is important to note that this is the case even for the capacity constrained interval $K_0 \leq K < K_1$. Thus if lenders are able to collude on monopoly prices, as modelled by Brock and Scheinkman (1985) for the one-segment case, they might as well be able to collude on price discrimination. If they collude on price discrimination, they will collude even for $K < K_1$. This is in contrast to Brock and Scheinkman (1985), where capacity constrained pricing equals competitive pricing. Furthermore, when the capacity constraint is not binding, the extra profit from price discrimination implies that lenders are able to collude on monopoly pricing even for a certain interval starting at $K_1$. This is also in contrast to Brock and Scheinkman (1985). These results are formulated in Proposition 3 in section 4.
For that proposition we need the term \( R = \Delta / (a_w - a)(a - c) \), which measures the collusive profit from monopolistic price discrimination as a fraction of the profit from deviation from monopolistic price discrimination\(^3\). As we will see in Proposition 3 price discrimination is sustainable if \( R \) is sufficiently large. In terms of the two-segment case illustrated in Figure 1, the \( R \) reduces to \( K_0 / (a - c) \). For a certain \( a_w \) and \( a_1 \), \( K_0 / (a - c) \) is an upper bound for \( R \). This illustrates the general role of \( K_0 \). The smaller is \( K_0 \), the smaller is the extra profit from price discrimination, and thus the larger is the incentive for deviation from price discrimination. We also note that \( R \) is always smaller than 1.

Finally, we will present the *rationing rule*, which applies when capacity is larger than the equilibrium demand. No lender will offer prices below \( c \). Lenders simultaneously and independently offer a price for every segment, and within every segment borrowers choose the offers having the lowest price, but they are not able to choose loans offered to other segments. In any one-segment equilibrium, i.e. where lenders offer one price for the full market, we apply a generalisation of the rationing rule in Brock and Scheinkman (1985), i.e.

\[
q_j = q(p_j|p_{-j}) = \max (0, (a - \sum_{i=1}^{j-1} k_i - p_j)k_j/\sum_{i=1}^{N} k_i),
\]

where \( p_j \) is a single price and \( p_{-j} \) is a vector of prices offered by the other lenders. Note that in case of a uniform price, the rule implies that the market is shared according to capacity. In a multi-segment equilibrium, we apply the Ben-Zvi and Helpman (1992) extensive structure of the stage game, i.e. where lenders first announce prices and then allocate capacities among segments. In Proposition 1 in their paper it is demonstrated that lenders will always agree on an allocation.

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\(^3\) As an illustration, the collusive profit at \( K_1 \) is \([(a - c)^2 + \Delta ]k_i/4K_1\), and the deviation profit is \((a_w - c)k_i/2\). The ratio of these two expressions is \([(a - c)^2 + \Delta ]/[(a - c)^2 + (a_w - a)(a - c)]\), where \( R = \Delta / (a_w - a)(a - c) \) thus is the ratio of the extra profits from price-discrimination for the two cases.
Applying this rationing rule, it is possible to identify the Nash equilibrium of the competitive stage game, as formulated in Proposition 1. The proposition combines previous results from Proposition 5 in Ben-Zvi and Helpman (1992), from Proposition 1 in Brock and Scheinkman (1985) and from Proposition 1 in Kreps and Scheinkman (1983). Recall that part 2b) of the proposition is for the multi-segment case only true for $K \geq K_0$, for smaller $K$ the optimal price will be at the partial linear aggregate demand curve.

**Proposition 1.**

1) There is no price discrimination in the stage game equilibrium.

2a) If $K - k_N \geq (a - c)$, then the only pure strategy Nash equilibrium is given by $p = c$, leading to $\pi^i = 0$.

2b) If $K + k_N \leq (a - c)$, then the only pure strategy Nash equilibrium is given by $p = a - K$, leading to $\pi^i = (a - K - c)k_i$.

2c) If $K - k_N < (a - c) < K + k_N$, then there is no pure strategy equilibrium. In the mixed strategy equilibrium the expected profit is $\pi^i = \pi^N \frac{k_i}{k_N}$, where $\pi^N = (a - c - K + k_N)^2/4$.

Below we will present the different cases described by the proposition. But first note that the equilibrium profits are continuous functions of $K$. This is obviously the case within each interval. At the critical value connecting case b) and c) we have the profit $\pi^i = k_i k_N$, while at the critical value connecting case c) and a) we have $\pi^i = 0$.

In contrast to Proposition 1 in Brock and Scheinkman (1985), the proposition is formulated such that it becomes transparent that aggregate capacity $K$ is the major determinant of equilibrium prices. Consequently the model can be applied to analyse shifts in the number of lenders (entry) as in Brock and Scheinkman (1985), or to analyse shifts in lending capacity as we do here.

We will not provide a proof of Proposition 1, since the proposition contains minor variations of previous results. Part 1 is proven by Ben-Zvi and Helpman (1992) for the capacity
constrained case b), where $K + k_N \leq (a - c)$. The generalisation to case a) and c) is intuitive. The argument is that in the competitive stage game, price discrimination is not sustainable due to lenders' incentive to allocate capacity to the highest priced segment⁴.

Since there is no price discrimination in the stage game, the equilibrium in part 2 is fully described by a one-segment model. Part 2 generalises Proposition 1 in Brock and Scheinkman (1985) to the case of heterogeneous capacities, and Proposition 1 in Kreps and Scheinkman (1983) to more than two players. In line with Brock and Scheinkman (1985) we refer to the proof in Kreps and Scheinkman (1983). We will now give the basic intuition behind part 2. Case c) is the most complex case, and the reader might prefer to consult Kreps and Scheinkman (1983).

Case a) is the excess capacity normal Bertrand case, where any set of $(N - 1)$ lenders can satisfy the maximal demand, and the equilibrium price will equal the marginal cost. Case b) is the normal capacity constrained case, where all lenders can profitably supply their full capacity, and the equilibrium price is the market-clearing price.

Case c) is an intermediate case, where the pure strategy equilibria from case a) and b) are not profitable, and a mixed strategy equilibrium has to be characterised. One of the possible pure strategies will be to overbid to meet the residual demand. This is more profitable for lender N than for the other lenders, since he will meet the largest residual demand, $q - K + k_N$. Thus lender N will name the single highest price with positive probability. In that case he names the price that is the best response when he meets the residual demand, $p_N = (a + c - K + k_N)/2$. In equilibrium the profit from this pure strategy, $\pi^*_N = (a - c - K + k_N)^2/4$, will equal his expected profit.

⁴ This is the outcome of the stage game, in the next sections we will see that price-discrimination may be sustainable in a repeated game.
The profit from his pure strategy lowest price $p_0$ also equals $\pi_N^n$. In the mixed strategy equilibrium he will in expectation sell his full capacity at this price, implying $(p_0 - c)k_N = \pi_N^n$. All other lenders will name an infinitesimal smaller price with positive probability, implying $(p_0 - c)k_i = \pi_i^n$. So there exist a lower bound for the equilibrium prices that no lender will name. If lender N names the price he will be undercut with positive probability. He rather names $p_0$, which is marginally above the lower bound, and lets the other undercut with positive probability. From these equilibrium profit functions, we have $(p_0 - c) = \pi_i^n/k_i = \pi_N^n/k_N$, and consequently $\pi_i^n = \pi_N^n k_i/k_N$.

3. The repeated one-segment case.

In this section we identify the equilibrium price for the one segment case as a function of aggregate lending capacity $K$. As shown in section 2, the model also covers the uniform price multi-segment case for $K \geq K_0$. This is because the aggregate inverse demand curve, $p = a - q$, will be identical. The one-segment case illustrates the structure of the multi-segment model of price discrimination in section 4. Except for heterogeneous capacities, the model in this section is the same as in Hatlebakk (2000), which in turn applies the Brock and Scheinkman (1985) model. The equilibrium outcome is formulated in Proposition 2, which is a special case of Proposition 3 in section 4. The present section also includes a comment on the empirical implications of the model, which is formulated in Corollary 1.

Proposition 1 describes lenders' equilibrium profit in the stage game. In this section the stage game is repeated infinitely. In case a collusive equilibrium is sustainable in the repeated game, we assume that lenders maximise joint profit, i.e. behave as a monopolist. We also assume that in case of collusion, the equilibrium demand is shared according to capacity, implying that collusive profit is shared according to capacity. So when collusion is sustainable, lenders share the joint profit,

$$\Pi^c = (p - c) (a - p),$$
leading to the individual collusive profit,

\[ \pi_i^c = (p - c) (a - p) k_i/K. \]

In any equilibrium a capacity constraint has to be satisfied, that is \( q = a - p \leq K \), and profit cannot be negative, that is \( p \geq c \). Finally, a deviation constraint has to be satisfied, which means that no lender will deviate in equilibrium. A lender who deviates will prefer to sell his full capacity at a price marginally below the equilibrium price, leading to the profit,

\[ \pi_i^d = (p - c) k_i. \]

We assume that lenders play trigger strategies, which means that in case of deviation, no lender will collude in any later period, and they will all have the stage game profit \( \pi_i^n \) from Proposition 1. Applying such strategies, Friedman (1971) proved that it is a subgame perfect Nash equilibrium to collude as long as the discounted value of collusion, evaluated at a uniform discount factor \( \delta \), is not less than the discounted value of deviation in period \( t = 0 \), and playing the stage game Nash equilibrium in all remaining periods\(^5\). Thus the deviation constraint becomes

\[ \pi_i^c - (1 - \delta) \pi_i^d - \delta \pi_i^n \geq 0, \]

which is equivalent to

\[ D = (p - c)(a - p) - (1 - \delta) (p - c)K - \delta \pi_i^n K/k_i \in 0. \]

Inserting for \( \pi_i^n \) from Proposition 1 we see directly, for case a) and b), that all lenders have the same incentive for deviation, that is \( k_i \) disappears from the expression. In case a) the

\[ 5 \text{ We can write this as } \sum_{i=0} \delta^i \pi_i^c \geq \pi_i^d + \sum_{i=1} \delta^i \pi_i^n, \text{ which is the same as } \frac{1}{1-\delta} \pi_i^c \geq \pi_i^d + \frac{\delta}{1-\delta} \pi_i^n, \text{ and consequently } \pi_i^c \geq (1-\delta) \pi_i^d + \delta \pi_i^n. \]
constraint reduces to $D/(p - c) = (a - p) - (1 - \delta) K \geq 0$. In case b) the constraint reduces to $D = (p - c)(a - p) - (1 - \delta)(p - c)K - \delta(a - c - K)K \in 0$.

In case c) the constraint becomes $D = (p - c)(a - p) - (1 - \delta)(p - c)K - \delta\pi^N_k K/k_N \geq 0$. This is the only case where the distribution of capacity matters. Note that as $K$ increases, it is likely that the individual capacity $k_N$ increases. We assume a linear relation defined by $K/k_N = \hat{N}$, where $\hat{N}$ is fixed. This simplifies the model. Note that the Brock and Scheinkman (1985) model of uniform capacities is the special case where $\hat{N} = N$. The assumption implies $D = (p - c)(a - p) - (1 - \delta)(p - c)K - \delta\pi^N_N K/k_N \geq 0$ for case c).

So we assume that if lenders are able to collude, they will collude on the price that maximises joint profit, subject to the deviation constraint, the capacity constraint, and the participation constraint. This means that the equilibrium is equivalent to the optimal choice for a monopolistic lender, subject to an additional deviation constraint, $D \geq 0$. This equilibrium is equivalent to the price that maximizes the Lagrange-function, $L = \Pi^* + \lambda_d D - \lambda_k (a - p - K) - \lambda_p (p - c)$, which is the same as,

$L = (1 + \lambda_d)(p - c)(a - p) - \lambda_d (1 - \delta)(p - c)K - \lambda_d \delta\pi^N_k K/k_i - \lambda_k (a - p - K) - \lambda_p (p - c)$.

In the proof of Proposition 2 we insert for $\pi^N_i$ from Proposition 1, and solve for the optimal price separately for the three cases a) to c). In Proposition 2 the equilibrium price function is identified for different intervals of $K$. The numbered cases refer to Proposition 3. The intermediate case c) from Proposition 1 is completely covered by case 6') in Proposition 2.

The proposition is illustrated in Figure 2. $K_4 = a - c - k_N$ is the lower bound and $K_5 = a - c + k_N$ is the upper bound for case c). The other $K$-s in the figure are defined in Proposition 2.

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6 The assumption that the price that maximise collusive profit is selected among all sustainable prices is a realistic assumption for infinitely repeated games, where we can imagine an adaptive game of equilibrium selection.
Recall that if the one-segment case is due to non-sustainable price discrimination, then part 1') is only true for $K \geq K_0 = a - a_1$.

Figure 2 about here.

The proposition is a representation of general results in the literature on folk theorems that followed the seminal paper by Friedman (1971). We identify the maximal price that is sustainable in collusion for a certain time preference factor $\delta$. For $K \leq K_3$ and $K \geq K_7$ only the competitive price is sustainable. For $K_3 \leq K \leq K_6$ the maximal collusive price is larger than the monopoly price, and the monopoly price is preferred. For the two intermediate intervals the maximal collusive price is determined by $\delta$ and $K$. Assumption A3 below makes sure that case c) is a sub-interval of $K_3 \leq K \leq K_6$. The assumption thus identifies a lower bound for $\delta$ that make sure that the monopoly price is sustainable for this sub-interval. A3 thus represents the term "sufficiently patient lenders" in the proposition. We have,

$$A3: \left( \frac{\hat{N} + 1}{2} \right) < \delta,$$

which is the same as $(1 - \delta) < (\hat{N} - 1)/2\hat{N}$.

Proposition 2 is a special case of Proposition 3, with the number of segments $w = 1$. For this special case, all results that are due to price discrimination become irrelevant, and assumptions A1 and A2 that will be presented for the general model are not needed here.
Proposition 2. (Proposition 3 applied to the one-segment case.)

Suppose sufficiently patient lenders who in case of collusion maximise joint profit and share the profit according to capacity, then we have the following equilibrium interest rate as a continuous function of $K$.

1') $p = a - K$, \quad if $K < (a - c)/(1 + \delta) = K_2$

5') $p = c + \delta K$, \quad if $K_2 \leq K \leq (a - c)/2 \delta = K_3$

6') $p = (a + c)/2$, \quad if $K_3 \leq K \leq (a - c)/(1 - \delta) = K_6$

7') $p = a - (1 - \delta)K$, \quad if $K_6 \leq K \leq (a - c)/(1 - \delta) = K_7$

8') $p = c$, \quad if $K_7 \leq K$

Proof: In the proof we identify the price that maximizes the Lagrange-function presented above. If no constraint is binding, that is $\lambda_d = \lambda_k = \lambda_p = 0$, then the function reduces to $\Pi^c = (p - c)(a - p)$, and the optimal unconstrained monopoly price is $p^* = (a + c)/2$, which implies the aggregate demand $q = (a - c)/2 = K_1$. This implies that whenever we prove that the unconstrained prices are sustainable, then $\lambda_k = 0$ for $K > K_1$. Below we will prove that $p^*$ is sustainable for case c), and consequently we do not have to prove $\lambda_d = \lambda_p = 0$ for case c).

Furthermore, the capacity constraint will never bind for $K > a - c$, and consequently we also have $\lambda_k = 0$ for case a).

Inserted for the unconstrained price $p^*$, we have $\Pi^c = (a - c)^2/4$. We insert for $\Pi^c$ in the deviation constraint $D^* = \Pi^c - (1 - \delta)(p - c)K - \delta \pi^*_i K/k_i$, which thus becomes a function of $K$. The function will differ between cases a) to c). Whenever $D^* \geq 0$, we will have $\lambda_d = 0$.

Below we insert for $\pi^*_i$ in the Lagrange function from Proposition 1, applying the assumption $K/k_N = \hat{N}$ for case c). We also insert zero for the Lagrange multipliers, for the cases discussed above.

Case a): $K - k_N \geq (a - c) \Leftrightarrow K \geq K_5$: The Lagrange function becomes $L = (1+\lambda_d)(p - c)(a - p) - \lambda_d (1 - \delta)(p - c)K - \lambda_p (p - c)$. We have $\lambda_d = \lambda_p = 0$ in equilibrium, as long as $D^* = (a -$
c) $(1 - \delta)(p - c)K \geq 0 \iff K \leq (a - c) / 2(1 - \delta) = K_6$. Thus the monopoly price $p^* = (a + c)/2$ is sustainable for $K_5 \leq K \leq K_6$.

For $K > K_6$ we have $D = (p - c)(a - p) - (1 - \delta)(p - c)K = 0 \iff p = a - (1 - \delta)K$. We have $\lambda_p = 0$, as long as $p = a - (1 - \delta)K > c$, i.e. the participation constraint does not bind as long as $p = a - (1 - \delta)K > c \iff K < (a - c)/(1 - \delta) = K_7$. Thus we have $p = a - (1 - \delta)K$ for $K_6 \leq K \leq K_7$, and $p = c$ for $K \geq K_7$.

Case b): $K + k_N \leq (a - c) \iff K \leq K_4$: The Lagrange function becomes $L = (1 + \lambda_d)(p - c)(a - p) - \lambda_d(1 - \delta)(p - c)K - \lambda_d(a - K - c)K - \lambda_k(a - p - K) - \lambda_k(p - c)$. We know that $K_1$ is the optimal monopoly quantity. For $K \leq K_1$ lenders maximise $\Pi^c$ by lending at full capacity, implying $p = a - K$, which equals the monopoly price for $a - K = p^* = (a + c)/2 \iff K = (a - c)/2 = K_1$.

For $K \geq K_1$, suppose that $p^*$ is sustainable, and thus $\lambda_d = \lambda_p = \lambda_k = 0$ Then $D^* = (a - c)^2/4 - (1 - \delta)(a - c)K/2 - \delta(a - K - c)K \geq 0 \iff (K - K_1)(K - K_3) \geq 0$, where $K_3 = (a - c)/2\delta$. The inequality is true for $K \geq K_3$, and $p^*$ is thus sustainable for $K_3 \leq K \leq K_4$.

For the interval $K_1 \leq K \leq K_3$, we have $D = (p - c)(a - p) - (1 - \delta)(p - c)K - \delta(a - K - c)K = 0 \iff (K - a + p)(\delta K + c - p) = 0$. For $K = K_2 = (a - c)/(1 + \delta)$ both parentheses equal zero. For $K_1 \leq K \leq K_2$, the competitive price $p = a - K$ is the most profitable. For $K_2 \leq K \leq K_3$, the constrained collusive price $p = c + \delta K$ is the most profitable.

Case c): $K - k_N < (a - c) < K + k_N \iff K_4 \leq K \leq K_5$: The Lagrange function becomes $L = (1 + \lambda_d)(p - c)(a - p) - \lambda_d(1 - \delta)(p - c)K - \lambda_d(a - c - K(\hat{N} - 1)/\hat{N})^2\hat{N}/4$. We have $D^* = (a - c)^2/4 - (1 - \delta)(a - c)K/2 - \delta(a - c - K(\hat{N} - 1)/\hat{N})^2\hat{N}/4 \geq 0$. We have $K_4 = (a - c)\hat{N}/(\hat{N} + 1)$ and $K_5 = (a - c)\hat{N}/(\hat{N} - 1)$. Consequently $K_4 > K_3 \iff (a - c)\hat{N}/(\hat{N} + 1) > (a - c)/2\delta \iff \delta > (\hat{N} + 1)/2\hat{N}$, and $K_5 < K_6 = (a - c)/2(1 - \delta) \iff \delta > (\hat{N} + 1)/2\hat{N}$, which for both cases are true by A3.
From case b) we know that the monopoly price \( p^* \) is sustainable at \( K_4 \) and from case a) we know that \( p^* \) is sustainable at \( K_5 \). We thus have \( D^* > 0 \) at \( K_4 \) and \( K_5 \). Furthermore, we have
\[
\partial D^*/\partial K = -(1 - \delta)(a - c)/2 + \delta ((a - c)(\hat{N} - 1) - K(\hat{N} - 1)^2/\hat{N})/2 \text{ and } \partial^2 D^*/\partial K^2 = -((\hat{N} - 1)^2/2\hat{N} < 0, \text{ which means that } D^* \text{ is concave, which in turn implies that } D > 0 \text{ for the full interval defining case } c). \]

Recall that the proposition is a variant of the Brock and Scheinkman (1985) model, which is illustrated in their Figure 3. Since they focus on entry, they have the interest rate as a function of \( N \), while we apply the general formulation, i.e. where aggregate capacity \( K \) determines the interest rate.

We will give the basic intuition behind the proposition that is illustrated in Figure 2. For \( K \leq K_1 \), the competitive price and the monopoly price coincide, and they are determined by the aggregate capacity. For \( K_1 \leq K \leq K_2 \) lenders would prefer to charge the monopoly price \( p^* \), but deviation profit is so high that only the capacity constrained competitive price is sustainable. As \( K \) increases within this interval the competitive profit decreases more than the deviation profit increases, and at \( K_2 \) it eventually becomes profitable to collude on a price above the competitive price. As \( K \) increases in the interval \( K_2 \leq K \leq K_3 \), the competitive profit decreases even more compared to deviation profit. Since the competitive profit applies as the punishment after a deviation, it becomes relatively less profitable to deviate. As a consequence, a higher collusive price is sustainable. For \( K \geq K_3 \) even the monopoly price \( p^* \) is sustainable for a large interval of \( K \). For \( K = K_4 \) the competitive profit becomes zero. The deviation profit still increases with \( K \), and for \( K \geq K_6 \) it eventually becomes profitable to deviate from \( p^* \). In that case the collusive price has to decline as \( K \) increases, to reduce the incentive for deviation. The price-reduction continues until \( p \) equals the marginal cost at \( K_7 \). For \( K \geq K_7 \) profit thus equals zero.
Note the part of the price-function where the interest rate increases with aggregate lending capacity. As explained above, the explanation is that the punishment profit decreases faster than the deviation profit increases in this range, and thus a higher collusive price is sustainable. This result is due to Brock and Scheinkman (1985).

However, it turned out to be difficult to identify an increasing part in our empirical study in Hatlebakk (2000). The remaining part of this section is to document that we cannot expect to identify the increasing part in empirical studies. This is because the interval from $K_1$ to $K_3$ is small compared to the interval from zero to $K_1$. This in turn means that even for a relatively small shift in $K$ at $K_1$, the monopoly price is sustainable because the new level of $K$ is larger than $K_3$. The basis for the conclusions are stated in Corollary 1.

We compare the relative sizes of the different intervals in Figure 2, and start out with a full description of the $K$-s as functions of $K_1$. We have $K_2 = 2K_1/(1 + \delta)$, $K_3 = K_1/\delta$, $K_4 = a - c - k_N = 2K_1 - K_4/\hat{N}$, which implies $K_4 = 2\hat{N}K_1/(\hat{N} + 1)$, $K_5 = a - c + k_N = 2K_1 + K_5/\hat{N}$, which implies $K_5 = 2\hat{N}K_1/(\hat{N} - 1)$, $K_6 = K_1/(1 - \delta)$, and $K_7 = 2K_1/(1 - \delta)$. Note that $K_4$ and $K_5$ are defined for different levels of $K$ and thus $k_N$. This means that $K_4$ and $K_5$ cannot be directly compared when stated as $K_4 = a - c - k_N$ and $K_5 = a - c + k_N$. But since $\hat{N} = K/k_N$ is fixed, $K_4$ and $K_5$ can be compared when stated as $K_4 = 2\hat{N}K_1/(\hat{N} + 1)$ and $K_5 = 2\hat{N}K_1/(\hat{N} - 1)$. To simplify the notation in the corollary, we denote $i = (1 - \delta)/\delta$, for the time-preference rate.

**Corollary 1.**

a) $K_1/K_3 = \delta$, the time-preference factor.

b) $K_3/K_6 = (K_3/K_1)/(K_6/K_1) = (1 - \delta)/\delta = i$, the time-preference rate.

c) $(K_6 - K_3)/(K_3 - K_1) = 1/i^2 - 1$.

**Proof:** Case a) and b) follows directly from the definitions. Case c): $(K_6 - K_3)/(K_3 - K_1) = [K_1/(1 - \delta) - K_1/\delta]/[K_1/\delta - K_1] = [\delta/(1 - \delta) - 1]/[1 - \delta]$. Inserting $\delta = 1/(1 + i)$, the ratio becomes $(1 - i)(1 + i)/i^2 = (1 - i^2)/i^2 = 1/i^2 - 1$. ||
Corollary 1 identifies the relative size of the different intervals of the price-function in Figure 2. Part a) identifies the length from zero to the monopoly quantity $K_1$ as a fraction of the length from zero to $K_3$, where the fraction equals the time-preference factor for the colluding lenders. Thus the fraction indicates the size of the deviation constrained interval relative to the capacity constrained interval. Part b) identifies the length from zero to $K_3$ as a fraction of the length from zero to $K_6$, where the fraction equals the time-preference rate for the colluding lenders. Thus the fraction indicates the size of the constrained interval relative to the unconstrained interval. Part c) identifies the length from $K_3$ to $K_6$ relative to the length from $K_1$ to $K_3$. Thus the ratio indicates the size of the unconstrained interval relative to the deviation constrained interval.

Note that all rates identified in Corollary 1 are determined by the time-preference rate of the colluding firms. Although our focus is on informal lenders, Corollary 1 is a general comment on the relevance of the counter-intuitive increasing part of the price-function identified by Brock and Scheinkman (1985). For developed economies, a time-preference rate of 0.1 is realistic. In that case the interval from $K_1$ to $K_3$ is about 10% of the interval from zero to $K_3$, and about 1% of the interval from $K_3$ to $K_6$. This means that if we expect capacity to vary over the full interval from zero to $K_6$, or even to $K_7 = 2*K_6$, which is the critical value for the ordinary Bertrand case, then the deviation constrained interval is very small. Furthermore, an increase in $K$ by more than 10% from $K_1$, will sustain the monopoly price. In case of entry in a model of homogenous firms, which is the focus of Brock and Scheinkman (1985), this is equivalent to an increase from 10 to 11 firms. Thus, we conclude that the counter-intuitive increasing part of the price-function is of marginal interest. As we will see in the next section, this conclusion becomes even stronger in the general model of price discrimination. However, we will emphasize that the interval of an unconstrained monopoly price is very large, which is thus the major result in Brock and Scheinkman (1985).
In less developed economies the time-preference rate might be higher than in developed economies. This is obviously the case for informal borrowers. In for example Nepal, informal borrowers pay interest rates in the range from the formal rate of 0.1 to more than 0.5. Thus even some of the informal lenders might have fairly high time-preference rates. However, the segmentation of the credit market indicates lending even at informal rates down towards 0.1. So a uniform time-preference rate for the colluding lenders is likely to be closer 0.1 than to 0.5. Figure 2 approximates a time-preference rate of 0.25, and we specify \( \hat{N} = 3 \) to identify \( K_4 \) and \( K_5 \). In that case we have \( K_2 = 1.1*K_1 \), \( K_3 = 1.25*K_1 \), \( K_4 = 1.5*K_1 \), \( K_5 = 3*K_1 \), \( K_6 = 5*K_1 \), and \( K_7 = 10*K_1 \). Note that the relative size of the deviation constrained interval is larger than in the previous example, where the time-preference rate was 0.1. The interval from \( K_1 \) to \( K_3 \) is now 20% of the interval from zero to \( K_3 \), and about 7% of the interval from \( K_3 \) to \( K_6 \). So we need a 20% increase in capacity from \( K_1 \) to sustain the unconstrained monopoly price.

In specifying the range of a reasonable time-preference rate we mentioned the likely lower bound of 0.1 and an upper bound of 0.5. In credit markets the price \( p \) is measured on the same scale, which means that the formal interest rate of 0.1 is a lower bound for \( c \), and 0.5 is a likely value for \( a \). Applying these values, the monopoly price \( p^* \) becomes 0.3. If we continue with our example, we can solve for the equilibrium interest rate at \( K_2 \), which from Proposition 2 is \( p = a - K_2 = a - (a - c)/(1 + \delta) = 0.5 - 0.4/1.8 = 0.28 \). So by inserting reasonable values in the theoretical model, we conclude that the deviation constrained interval is relatively small, and if \( K \) turns out to be in this interval, the constrained price will be only marginally below the unconstrained monopoly price.

Corollary 1 has implications for empirical analyses of informal rural credit markets. In Hatlebakk (2000) we estimate a price-function as in Figure 1, using a proxy for village lending capacity \( K \) as one of the explanatory variables. The realistic example above indicates that we cannot expect to identify an increasing part of the price-function, even though Proposition 2 indicates that such an interval will exist.
Corollary 1 has another implication. Suppose that we can only expect to identify a decreasing and then a flat part of the estimated price-function. Then the flat part may instead represent marginal-cost pricing, and not necessarily monopoly pricing. Thus we need an alternative empirical test to discriminate between the stage game model of marginal-cost pricing and the repeated game model of monopoly pricing. As we know from section 2, price discrimination is not sustainable in the stage game. In the next section we will prove that price discrimination is sustainable in the repeated game. Consequently we have an alternative test for identification of the appropriate theoretical model.

4. The repeated multi-segment model.

In this section we present the general model, which has the model in section 3 as a special case. The presentation follows the presentation in section 3. The section can be read independently of section 3, but it might be useful to first read section 3, or to read the sections in parallel.

Proposition 1 describes lenders' equilibrium profit in the stage game. In this section the stage game is repeated infinitely. In case a collusive equilibrium is sustainable in the repeated game, we assume that lenders maximise joint profit, i.e. behave as a price-discriminating monopolist. We also assume that in case of collusion, demand from every segment is shared according to capacity, implying that collusive profit is shared according to capacity. So when collusion is sustainable, lenders share the joint profit,

\[ \Pi^c = \sum_{s=1}^{w} (p_s - c)(a_s - p_s) / b_s , \]

leading to the individual collusive profit,

\[ \pi^c_i = \sum_{s=1}^{w} (p_s - c)(a_s - p_s)k_i / K_b . \]
In any equilibrium a capacity constraint has to be satisfied, that is \( q = \sum_{s=1}^{w} (a_s - p_s) / b_s = a - \sum_{s=1}^{w} p_s / b_s \leq K \), and profit cannot be negative in any segment, that is \( p_1 \geq c \). Finally a deviation constraint has to be satisfied, which means that no lender will deviate in equilibrium. A lender who deviates will prefer to sell his full capacity at a price marginally below the equilibrium price in the highest priced segment. In case his capacity is larger than the demand in that segment, he will supply the rest in the segment having the second highest price, and so on. In the general case, the deviation profit thus becomes \( \pi_d = \sum_{s=1}^{w} (p_s - c) \max[0, \min(q_s(p_s), k_i - \sum_{s=v+1}^{w} q_v(p_v))] \), where \( q_s(p_s) = (a_s - p_s)/b_s \). However, to simplify the model we assume that no lender is able to fully serve the highest priced segment, which in turn means that this segment is relatively large. In case the highest priced segments are actually small, the simplification means to consider many small segments as a joint segment\(^7\). The simplification implies that we apply an upper bound for the deviation profit. Thus, the deviation profit equals

\[
\pi_d = (p_w - c)k_j.
\]

We assume that lenders play trigger strategies, which means that in case of deviation, no lender will collude in any later period, and they will all have the stage game profit \( \pi^n \) from Proposition 1. Applying such strategies, Friedman (1971) proved that it is a subgame perfect Nash equilibrium to collude as long as the discounted value of collusion, evaluated at a uniform discount factor \( \delta \), is not less than the discounted value of deviation in period \( t = 0 \), and playing the stage game Nash equilibrium in all remaining periods. Thus the deviation constraint becomes \( \pi^c - (1 - \delta) \pi_d - \delta \pi^n \geq 0 \), which is equivalent to

\[
D = \Pi^c - (1 - \delta) \pi_d K / k_i - \delta \pi^n K / k_i = \sum_{s=1}^{w} (p_s - c)(a_s - p_s) / b_s - (1 - \delta) (p_w - c)K - \delta \pi^n K / k_i \leq 0.
\]

\(^7\) In Nepal there are some few focal interest rates, which means that it is reasonable to consider small segments as a part of a larger segment. It is for example relatively few borrowers who pay interest rates above 0.5, and they might be considered as included in the large segment paying 0.5.
Inserting for \( \pi_i^a \) from Proposition 1 we see directly for case a) and b), that all lenders have the same incentive for deviation, that is \( k_i \) disappears from the expression. In case c) the constraint becomes

\[
D = \sum_{s=1}^{w} (p_s - c)(a_s - p_s) / b_s - (1-\delta)(p_w - c)K - \delta \pi_i^a K/k_N \geq 0.
\]

This is the only case where the distribution of capacity matters. Note that as \( K \) increases, it is likely that \( k_N \) increases. We assume a linear relation defined by \( K/k_N = \hat{N} \), where \( \hat{N} \) is fixed. This simplifies the model. Note that in case of uniform capacities we have \( \hat{N} = N \). The assumption implies

\[
D = \sum_{s=1}^{w} (p_s - c)(a_s - p_s) / b_s - (1-\delta)(p_w - c)K - \delta(a - c - K(\hat{N} - 1)/\hat{N})^2 \hat{N} /4 \geq 0.
\]

So we assume that if lenders are able to collude, they will collude on the price that maximise joint profit, subject to the deviation constraint, the capacity constraint, and the participation constraint. This means that the equilibrium is equivalent to the optimal choice for a monopolist lender, subject to an additional deviation constraint, \( D \geq 0 \). This equilibrium is equivalent to the price-vector that maximizes the Lagrange-function,

\[
L = \Pi_c + \lambda_d D - \lambda_k (a - \sum_{s=1}^{w} p_s / b_s \leq K) - \lambda_p (p_1 - c),
\]

which is the same as,

\[
L = (1+\lambda_d) \sum_{s=1}^{w} (p_s - c)(a_s - p_s) / b_s - \lambda_d (1-\delta)(p_w - c)K - \lambda_d \delta \pi_i^a K/k_i - \lambda_k (a - \sum_{s=1}^{w} p_s / b_s \leq K) - \lambda_p (p_1 - c).
\]

In the proof of Proposition 3 we insert for \( \pi_i^a \) from Proposition 1, and solve for the optimal prices separately for the three cases a) to c). In Proposition 3 the equilibrium price functions are identified for different intervals of \( K \), which are numbered from 1 to 8. The intermediate case c) from Proposition 1 is completely covered by case 6) in Proposition 3.

The proposition is illustrated in Figure 3. The \( K_4 = a - c - k_N \) is the lower bound and \( K_5 = a - c + k_N \) is the upper bound for case c). The other \( K \)-s in the figure, are defined in Proposition 3. All \( \hat{K}_i \) in Proposition 3 will reduce to the corresponding \( K_i \) when \( w = 1 \), and the proposition
reduces to Proposition 2 in that case. Recall, from section 1 that we simplify the model by assuming \( K \geq K_0 \). We still draw the likely price-function for \( K < K_0 \) in Figure 3.

We apply assumptions A1 to A3. A3 is applied for the same purpose as in the one-segment model, that is to simplify the proof of case c). The assumption identifies a lower bound for the time-preference factor \( \delta \), which implies that the monopoly price is sustainable even for this sub-interval.

For the multi-segment case we add A2, which is also a restriction on the time-preference factor. A2 implies that collusive price discrimination is sustainable. The term \( R \) is described in more detail in section 2, and represents the profit from equilibrium price discrimination relative to the profit of deviation from price discrimination.

In principle only one of assumptions A2 and A3 may be satisfied, and as such they are stated separately. If only A3 is satisfied, then we will have a uniform price as in section 3, where we can still have collusion. If only A2 is satisfied, then it will not be sure that collusion is sustainable for case c). A3 is the same as \( (1 - \delta) < (\hat{N} - 1)/2\hat{N} \). With A3 at this form we may argue that for realistic parameter values, A3 is likely to be an implication of A2.

A1 is to simplify the proof of case b). It implies that the multi-segment version of \( K_2 \) is to the left of \( K_3 \). This is in turn applied to prove that the multi-segment version of \( K_3 \) is to the left of \( K_3 \). Recall the assumption \( a_1 > (a + c)/2 \) from section 2. In combination with A1 this implies the necessary condition \( 4a_1 > a_w + 3c \).
Assumptions.

A1: \( a > (a_w + c)/2 = p^*_w \)

A2: \( R \equiv \Delta / (a - c)(a_w - a) > (1 - \delta)/\delta \), the time preference rate.

A3: \( (\hat{N} + 1)/2 \hat{N} < \delta \), the time preference factor.

To make the proof more transparent we will apply A4 and A5. A4 is an obvious implication of A2. A5 is also an implication of A2, since \( K_0(a_w - a)(1 + \delta)/\delta > \Delta \) is always true.

A4: \( R \equiv \Delta / (a - c)(a_w - a) > (1 - \delta) \)

A5: \( (a - a_1)/(a - c) > (1 - \delta)/(1 + \delta) \)

In the proof we also need the fact \( R < 1 \), see section 2.

Proposition 3.

Suppose sufficiently patient lenders who in case of collusion, maximise joint profit and share the profit according to capacity, then we have the following vector of equilibrium interest rates, where the interest rates are continuous functions of \( K. \)

1) \( p_s = (a_s + a)/2 - K, \forall s, \) if \( K_0 = a - a_1 < K < (a - c)/2 = K_1, \)

2) \( p_s = (a_s + c)/2, \forall s < w, \) if \( p_s < p_w \) and \( K_1 \leq K \leq (a - c)/(1 - \delta) = K_7, \)

3) \( p_w = (a_w + c)/2, \) if \( K_1 \leq K \leq \hat{K}_1, \) where \( \hat{K}_1 > K_1. \)

4) \( \partial p_w / \partial K < 0, \) if \( \hat{K}_1 \leq K < \hat{K}_2, \)

5) \( \partial p_w / \partial K > 0, \) if \( \hat{K}_2 < K \leq \hat{K}_3, \) where \( \hat{K}_3 \leq (a - c)/2 \delta = K_3, \)

6) \( p_w = (a_w + c)/2, \) if \( \hat{K}_3 \leq K \leq \hat{K}_6 \leq (a - c)/(1 - \delta) = K_6, \)

7) \( \partial p_w / \partial K < 0, \) if \( \hat{K}_6 \leq K < (a - c)/(1 - \delta) = K_7, \)

8) \( p_s = c, \forall s, \) if \( K_7 \leq K. \)

\(^8\) From section 2 we recall that \( K_0(a - c) \) is an upper bound for \( R, \) and thus \( K_0(a_w - a) > \Delta, \) which in turn implies \( K_0(a_w - a)(1 + \delta)/\delta > \Delta. \)
The proof is in the appendix. We have included the one-segment equilibrium price-function from Figure 2 in Figure 3 for comparison. In the multi-segment case we will have a set of parallel price-functions representing the equilibrium price-vector. To simplify the figure we have only included the price-function for segment w, which thus can be compared to the uniform price from the one-segment case. The lower line in the figure is thus not representing another segment. If we had included a line for another segment, it would have been horizontal from $K_1$ until it meets the $p_w$ line.

Figure 3 about here.

We will give the basic intuition behind Proposition 3 by referring to Figure 3. For $K \leq K_1$, the capacity constraint is binding. In contrast to the one-segment model, monopolistic price discrimination is sustainable even for this case. Price discrimination means to allocate capacity among the segments according to the marginal income. As $K$ decreases in the interval, the equilibrium marginal income increases, and eventually the lowest priced segment will have zero demand. This happens at $K_0/2$, where the $p_w$-function bends\(^9\). This process continues until only segment w is left.

For the interval $K_1 \leq K \leq K_3$, we recall that the deviation constraint was binding for the one-segment case. In the general case this is not necessarily true due to the extra profit from price discrimination. If the constraint binds, it will not bind immediately at $K_1$, and it will bind only for a sub-segment of $K_1 \leq K \leq K_3$. This is because the extra profit from price discrimination from the interval below $K_1$ is present also for intervals above $K_1$.

---

\(^9\) Recall that for a uniform price demand equals zero when $p = a - K = a_1$. In the multi-segment case demand equals zero when the aggregate marginal income curve (the horizontal sum of the marginal income curves) equals $a_1$, that is when $a - 2K = a_1$. 
For a large interval the unconstrained monopoly price-vector is sustainable. The lower end of the interval is to the left of $K_3$ and the upper end is between $K_5$ and $K_6$. In the one-segment case it is exactly the interval from $K_3$ to $K_6$. At the upper end the deviation constraint binds, and it will bind for a large interval. In that interval the collusive prices will decrease as lending capacity increases to counter-act the stronger incentive for deviation. For any $K$, only the highest priced segment is affected, and lenders charge the unconstrained monopoly price in the other segments. However, as $K$ increases in the interval, $p_w$ will eventually equal the unconstrained $p^*_{w-1}$ and the two segments merge. This continues until all segments are merged, and we have the one segment case. Consequently for $K \geq K_7$ the uniform price $p$ equals the marginal cost.

Note that price discrimination makes collusion easier in a specific sense. If the time-preference factor satisfies A2 and $K < K_2$, then collusion is sustainable only for the multi-segment case and not for the one-segment case.

5. Conclusion.

In Hatlebakk (2000) we found empirical support for capacity constrained oligopolies in the informal rural credit markets of Nepal. The theoretical model applied in that paper is a version of Brock and Scheinkman (1985). Two important characteristics of informal rural credit markets were not covered by the model, that is, heterogeneous lending capacities and segmentation of the demand side. We were not fully satisfied with the empirical results, as long as the theoretical model did not reflect these characteristics. A more general model might lead to an alternative empirical specification.

The present paper generalises the Brock and Scheinkman (1985) model. The general model is in line with the empirical analysis of informal rural credit markets in Hatlebakk (2000). In contrast to Brock and Scheinkman (1985), collusive prices are sustainable even for the capacity constrained case, due to price discrimination. As in the Brock and Scheinkman
model unconstrained monopoly prices are sustainable in tacit collusion for a large interval of aggregate capacity. For a relatively small interval, see Corollary 1, the collusive prices are constrained due to the incentive for deviation. The deviation constrained interval is even smaller in the multi-segment case than in the one-segment case.

The small constrained interval implies that the counter-intuitive increasing part of the price-function in Brock and Scheinkman (1985) might be difficult to identify in empirical analysis. Consequently, an empirical specification with two linear parts of the price-function may be sufficient, i.e. a decreasing part along the demand curves, and a flat part in line with the unconstrained monopoly prices. Empirical support for two linear parts is also in line with the stage game capacity constrained Bertrand model, i.e. where the flat part represents marginal cost. Additional empirical analyses are needed to discriminate between the two models. In Hatlebak (2000) we find support for monopoly pricing but not for cost pricing.
Appendix. Proof of Proposition 3.

In the proof we identify the price-vector that maximizes the Lagrange-function,

\[ L = (1+\lambda_d)\sum_{s=1}^{W} \frac{(p_s - c)(a_s - p_s)}{b_s} - \lambda_d (1 - \delta) (p_w - c)K - \lambda_k \delta \pi^n K/k_i - \lambda_k (a - \sum_{s=1}^{W} p_s / b_s \leq K) - \lambda_p (p_1 - c). \]

If no constraint is binding, that is \( \lambda_d = \lambda_k = \lambda_p = 0 \), then the function reduces to \( \Pi^c = \sum_{s=1}^{W} \frac{(p_s - c)(a_s - p_s)}{b_s} \), and the optimal unconstrained monopoly prices are \( p^*_s = (a_s + c)/2 \), which implies the aggregate demand \( q = (a - c)/2 = K_1 \). This implies that whenever we prove that the monopoly prices are sustainable, then the capacity constraint is not binding for \( K > K_1 \), and we have \( \lambda_k = 0 \). Furthermore the participation constraint is not binding in that case, which implies \( \lambda_p = 0 \). Below we will prove that the monopoly prices are sustainable for case c), and consequently we do not have to prove \( \lambda_k = \lambda_p = 0 \) for case c). Furthermore, the capacity constraint will never bind for \( K > a - c \), and consequently we have \( \lambda_k = 0 \) for case a). In formulating the Lagrange functions below we insert zero for the Lagrange multiplicators in these cases.

From the Lagrange function we see that the deviation constraint will only affect \( p_w \), and the participation constraint will only affect \( p_1 \). However, when the deviation constraint is binding, we may have the case where the constrained \( p_w \) equals \( p^*_{w-1} \). Thus the two segments will have a uniform price, and we rename the merged segment as \( w \). The deviation constraint may thus bind for all segments, that is, when the segments have merged into one segment. So when the deviation constraint is binding the smallest possible price is the price from the one-segment model in section 3. This in turns means that the participation constraint is only binding for \( K \geq K_7 \), and we can simplify the proof by inserting \( \lambda_p = 0 \) for cases b) and c).

Inserted for the monopoly prices \( p^*_s \), we have \( \Pi^c = (\Delta + (a - c)^2)/4 \). We insert for \( \Pi^c \) in the deviation constraint, \( D^* = \Pi^c - (1 - \delta)(p_w - c)K - \delta \pi^n K/k_i \), which thus becomes a
function of \(K\). The function will differ between cases a) to c). The main strategy of the proof is to identify the intervals of \(K\) where \(D^* \geq 0\), which means that there is no incentive for deviation, and we have \(\lambda_d = 0\). For the intervals where \(D^* < 0\), we study the effect of a change in \(K\) on \(p_w\), subject to the constraint that \(D = 0\).

As in section 3 we insert for \(\pi^n_i\) from Proposition 1, and we apply the assumption \(K/k_N = \hat{N}\) for case c). We solve for the optimal prices separately for the three cases a) to c). The structure of the proof is the following, referring to cases a) to c) in Proposition 1.

For case a), we first identify the non-empty interval for \(K\) where \(D^* \geq 0\). Next we total differentiate \(D = 0\), to demonstrate that the constrained \(p_w\) decreases as \(K\) increases. Finally we document that when all segments are merged into \(w\), we will have \(p = c\) for \(K \geq K_7\).

For case b) we first consider the capacity constrained case, \(K \leq K_1\). We identify the capacity constrained optimal prices. Then we calculate \(D\), and document that \(D > 0\). Next we consider the case \(K \geq K_1\). We first document that \(D > 0\) at \(K_1\) and \(K_3\). Next we document that \(D^*\) is convex, and we identify \(K^*\) where \(D^*\) has its minimum. Then we document that \(K^* < K_3\), which implies that \(D > 0\) for any \(K > K_3\) for case b). Within the interval \(K_1\) to \(K_3\) we thus may have an interval where \(D < 0\). We document that if such an interval exists, then it will never start at \(K_1\). Finally we have to prove that if the interval exists, then the capacity constraint will not bind at the same time. We also document that the deviation constrained prices will first decrease and then increase.

For case c) we document that \(D > 0\) at both ends of the interval, and then we document that \(D^*\) is concave, and thus \(D > 0\) for the full interval.

Case a): \(K - k_N \geq (a - c) \iff K \geq K_5\): The Lagrange function becomes \(L = (1 + \lambda_d)\sum_{s=1}^n (p_s - c)(a_s - p_s) / b_s - \lambda_d (1 - \delta)(p_w - c)K - \lambda_p (p_1 - c)\). We have \(\lambda_d = \lambda_p = 0\) in equilibrium, as long as \(D^* = (\Delta + (a - c)^2)/4 - (1 - \delta)(p_w - c)K \geq 0 \iff K \leq (\Delta + (a - \Delta)\).
\(c^2) / 2(1 - \delta)(aw - c) = \hat{K}_6\), which reduces to 
\(K \leq (a - c) / 2(1 - \delta) = K_6\), for \(w = 1\). Thus we have monopoly prices in the interval from \(K_5\) to \(\hat{K}_6\).

This interval is not empty since 
\(\hat{K}_6 > K_5 \iff (\Delta + (a - c)^2)/(\hat{N} - 1) \leq (\Delta + (a - c)^2)(\hat{N} - 1)/2\hat{N}(a - c)(aw - c) < (1 - \delta)\). By \(A4\) we have \(\Delta/(a - c)(aw - a) > (1 - \delta)\), and by \(A3\) we have \((\hat{N} - 1)/2\hat{N} > (1 - \delta)\). Thus we have \(\hat{K}_6 > K_5 \iff [(\Delta + (a - c)^2)(\hat{N} - 1)/2\hat{N}(a - c)(aw - c) < \Delta + (a - c)^2 \Rightarrow \Delta/(a - c)(aw - a) = R < 1\), which is always true. So as in the one-segment we apply \(A3\) to make sure that the monopoly equilibrium is sustainable at \(K_5\).

The unconstrained interval is smaller than in the one-segment case since we have \(K_6 \geq \hat{K}_6 \iff (a - c) / 2(1 - \delta) \geq \sum_s (a_s - c)^2 / 2(1 - \delta)(aw - c)b_s \iff (a - c) \geq \sum_s (a_s - c)^2 / (aw - c)b_s\).

For \(K > \hat{K}_6\) we solve for the optimal price-vector from the Lagrange function, \(L = (1 + \lambda_d)[\sum_s (p_s - c)(a_s - p_s) / b_s] - \lambda_d (1 - \delta) (pw - c)K\), leading to \(\partial L / \partial p_s = (1 + \lambda_d)(a_s + c - 2p_s)/b_s = 0, \forall s \neq w\), and consequently the optimal \(p_s = (a_s + c)/2\), and \(\partial L / \partial p_w = (1 + \lambda_d)(aw + c - 2pw)/bw - \lambda_d (1 - \delta)K = 0\), and consequently the optimal \(p_w = (aw + c)/2 - (1 - \delta)bwK\lambda_d/2(1 + \lambda_d)\), and \(\partial L / \partial \lambda_d = 0\), and consequently the binding deviation constraint \(\sum_s (p_s - c)(a_s - p_s) / b_s = (1 - \delta)(pw - c)K\). Note that in this case, \(p_w\) is smaller than in the unconstrained case, while the other prices are the same as in the unconstrained monopoly case, so the deviation constraint only affects the highest price.

Inserting the optimal \(p_s, \forall s \neq w\), in the binding deviation constraint, we can in theory solve for the optimal \(p_w\), and thus recursively find the equilibrium \(\lambda_d\) from the \(p_w\)-function. However, note that the binding deviation constraint is an implicit function of \(p_w\) and \(K\). So, by total differentiation of the constraint we have \(dp_w/dK = (1 - \delta)(pw - c)bw/(aw + c - 2pw - (1 - \delta)bwK) < 0\), iff \(p_w > (aw + c)/2 - (1 - \delta)bwK/2\), which is always true when \(\lambda_d > 0\). So,
from an unconstrained price \( p_w = (a_w + c)/2 \) at \( K = \hat{K}_6 \), \( p_w \) reduces (to reduce the incentive for deviation) as \( K \) increases (which increases the incentive of deviation).

Segments are sorted according to price. This means that for some \( K > \hat{K}_6 \), we will have \( p_w = p_{w-1} \). Then, the two segments merge. Renaming the joint segment as \( w \), the proof can be repeated for any \( w > 1 \), meaning that the maximum price will continuously decrease as \( K \) increases, until all segments are merged. In this one-segment case, the optimization problem will be the same as in section 3; \( \max (p - c)(a - p) \) s.t. \( (p - c)(a - p) \geq (1 - \delta)(p - c)K \), where the constraint reduces to \( p \leq a - (1 - \delta)K \). We know that the deviation constraint is binding, leading to the constrained optimal \( p = a - (1 - \delta)K \). The participation constraint does not bind as long as \( p = a - (1 - \delta)K > c \iff K < (a - c)/(1 - \delta) = K_7 \).

**Case b):** \( K + k_N \leq (a - c) \iff K \leq K_4 \): The Lagrange function becomes \( L = (1+\lambda_d) \sum_{s=1}^{w} (p_s - c)(a_s - p_s)/b_s - \lambda_d (1 - \delta) (p_w - c)K - \lambda_d \delta (a - K - c)K - \lambda_s (a - \sum_{s=1}^{w} p_s /b_s \leq K) \).

Suppose the deviation constraint does not bind when the capacity constraint binds, then lenders maximise profit subject to the binding capacity constraint, i.e. lenders maximise \( \sum_{s=1}^{w} (a_s - b_s q_s - c)q_s \), s.t. \( \sum_{s=1}^{w} q_s = K \), by allocating \( K \) among segments such that marginal income is equalised, leading to optimal constrained prices, \( p_s = (a_s + a)/2 - K \), which equals the unconstrained prices when \( (a_s + a)/2 - K = (a_s + c)/2 \iff K = (a - c)/2 = K_4 \). So, \( K_4 \) is the maximal \( K \) for which the capacity constraint binds. The capacity constrained prices imply the constrained quantities \( q_s = \frac{(a_s - a)}{2} + \frac{K}{b_s} + \frac{K^2}{2b_s} + \frac{Kc}{b_s} + \frac{a_s - a}{4b_s} - \frac{(a_s - a)K}{2b_s} - \frac{(a_s - a)c}{2b_s} \).

Inserting for the constrained \( p_w \), the deviation constraint does not bind iff \( D = (a - K - c)K + \Delta/4 - (1 - \delta)((a_w + a)/2 - K - c)K - \delta (a - K - c)K \geq 0 \iff (1 - \delta)(a - K - c)K + \Delta/4 > (1 - \delta) \)
\[(a_w + a)/2 - K - c)K \Leftrightarrow \Delta/2 > (a_w - a)K(1 - \delta).\] Inserting for the critical (largest) value \(K_1\) for the interval, the condition becomes \(\Delta/2 > (1 - \delta)(a - c)(a_w - a)/2 \Leftrightarrow R > (1 - \delta),\) which is true by A4.

This statement is only true for \(K \geq K_0.\) For smaller \(K\) the parameter \(a\) in stage game profit-function becomes larger, and for \(K < K_0/2\) the collusive profit is not precisely described by \((a - K - c)K + \Delta/4,\) see the introduction. Thus price discrimination is not necessarily sustainable in this interval. For very small \(K,\) it is definitely not sustainable, since all demand will be allocated to segment \(w.\) The proposition is thus only stated for \(K \geq K_0.\)

For \(K \geq K_1,\) the capacity constraint is not binding for unconstrained prices. It is potentially binding for deviation constrained prices, which we will check. However, we first identify the segment \(\hat{K}_1 \leq K \leq \hat{K}_3,\) where the deviation constraint is binding. Inserting the unconstrained prices we have \(D^* = ((a - c)^2 + \Delta)/4 - (1 - \delta)((a_w - c)/2)K - \delta(a - K - c)K \geq 0 \Leftrightarrow D^* = \delta(K - K_1)(K - K_3) - (1 - \delta)K(a_w - a)/2 + \Delta/4 \geq 0,\) where \(K_3 = (a - c)/2\delta.\) Inserting for \(K_1,\) we have \(D^*(K_1) = \Delta/4 - (1 - \delta)(a - c)(a_w - a)/4 > 0 \Leftrightarrow (1 - \delta) < R,\) which is true by A4. Inserting for \(K_3\) we have \(D^*(K_3) = \Delta/4 - (1 - \delta)(a - c)(a_w - a)/4 \delta > 0 \Leftrightarrow (1 - \delta)/\delta < R,\) which is true by A2. So, we have \(D^* > 0\) at \(K_1 \text{ and } K_3.\)

Next we have \(\partial D^*/\partial K = - (1 - \delta)(a_w - c)/2 - \delta(a - c) + 2\delta K - (1 - \delta)(a_w - c)/2 - \delta(a - c) = 0 \Leftrightarrow K^* = K_1 + (1 - \delta)(a_w - c)/4\delta.\) So, at \(K^*\) we have the optimum of \(D^*.\) We also have \(K^* = K_3 - (1 - \delta)(a - p^*_w)/2\delta.\) By A1 we have \(K^* < K_3,\) and by \(\partial^2 D^*/\partial K^2 = 2\delta > 0,\) \(D^*\) is convex. Since \(K^* < K_3,\) and \(D^* > 0\) at \(K_3\) and \(K_1,\) any interval of \(K\) having \(D^* < 0,\) has to be a sub-interval of the interval \(K_1 < K < K_3.\) This interval, \(\hat{K}_1 \leq K \leq \hat{K}_3,\) is not necessarily empty.

We will now characterise the interval \(\hat{K}_1 \leq K \leq \hat{K}_3.\) Note that for any pair of \(K\) having the same value for \(D^*,\) the sum of the two \(K\)-s equals \(2K^*.\) This is also true for the pair \(\hat{K}_1\) and \(\hat{K}_3\) where \(D^* = 0,\) i.e. we have \(\hat{K}_1 + \hat{K}_3 = 2K^*,\) and thus \(\hat{K}_1 - K_1 = K_3 - \hat{K}_3 + (1 - \delta)(a_w - \)
a)/2δ, i.e. the distance $\hat{K}_1 - K_1$ is larger than $K_3 - \hat{K}_3$. This is because $D^*$ starts out with a positive value at $K_1$, and although it may become negative, it will not happen immediately.

Suppose now that the interval $\hat{K}_1 \leq K \leq \hat{K}_3$ is not empty, then we have to prove that the capacity constraint will not bind when $D = 0$. We insert the unconstrained demand for all segments except for $w$ in the capacity constraint, leading to $p_w \geq (a_w + c)/2 + b_w(K_1 - K) = \bar{p}_w$, which is a function of $K$. Similarly we can insert the unconstrained prices in the other segments in $D$, leading to $D = ((a - c)^2 + \Delta_w)/4 - (2p_w - a_w - c)^2/4b_w - (1 - \delta)(p_w - c)K - \delta(a - K - c)K$. We use the subscript $w$ for $\Delta$ to remind the reader that as $p_w$ reduces, segments merge, and consequently $\Delta_w$ reduces. If we now insert $\bar{p}_w$ in the $D$-function, we have $D = ((a - c)^2 + \Delta)/4 - (2\bar{p}_w - a_w - c)^2/4b_w - (1 - \delta)(\bar{p}_w - c)K - \delta(a - K - c)K = ((a - c)^2 + \Delta)/4 + b_w\delta(K - K_1)(K_3 - K) + \delta K(p_w - a + K)$. Only the last part is potentially negative, and this part is not negative if $K > a - p_w = a - (a_w + c)/2$. Inserting for $K_1$, which is smaller than any $K$ in the interval, we have the critical condition $(a - c)/2 > a - (a_w + c)/2 \iff a_w > a$, which is true as long as there is more than one segment left.

To prove that the capacity constraint will not bind, it is thus sufficient to prove that all segments will not merge for $D = 0$. If they did, there would exist a critical $K$ where $p_2 = p^*_1 = (a_1 + c)/2 = p = a - K$, i.e. where the constrained price in segment 2 equals the unconstrained price in segment one, and consequently equals a uniform capacity constrained price. This condition imply the critical $K = a - (a_1 + c)/2$, which by A5 is larger than $K_2$, and is thus outside the capacity constrained interval for the uniform price case. So, the capacity constraint will never bind for $D = 0$.

We thus know that $D^*$ might be negative for a non-empty interval $\hat{K}_1 \leq K \leq \hat{K}_3$. It is reasonable to expect that $p_w$ will be a continuous function of $K$ in the interval. The remaining part of the proof for case b) is to prove that this is the case.
We have $D = ((a - c)^2 + \Delta)/4 - (2p_w - a_w - c)^2/4b_w - (1 - \delta)(p_w - c)K - \delta(a - K - c)K = 0$. Note that $\partial D/\partial p_w = D'_p = (a_w + c - 2p_w)/b_w - (1 - \delta)K$, and define the function $p'_w = (a_w + c)/2 - (1 - \delta)b_wK/2$, as the prices where $D'_p = 0$. Then we have $D'_p \begin{cases} > 0 \text{ iff } p_w < p'_w \\ = 0 \text{ iff } p_w = p'_w \text{. At } \hat{K}_1, \\ < 0 \text{ iff } p_w > p'_w \end{cases}$ we know that $p_w$ equals the optimal unconstrained price, which is larger than $p'_w$, and consequently $D'_p < 0$. Next, note that $\partial D/\partial K = - (1 - \delta)p_w + c - \delta(a - 2K)$, and define the function $p^k_w = (c - \delta a)/{(1 - \delta)} + 2\delta K/(1 - \delta)$, as the prices where $D'_K = 0$. Then we have $D'_K \begin{cases} > 0 \text{ iff } p_w < p^k_w \\ = 0 \text{ iff } p_w = p^k_w \text{. At } \hat{K}_1, \\ < 0 \text{ iff } p_w > p^k_w \end{cases}$

$\delta a)/(1 - \delta) + 2\delta \hat{K}_1/(1 - \delta) \iff 2\hat{K}_1 < (1 - \delta)(a_w - a)/2\delta + K_3 + K_1 = \hat{K}_1 + \hat{K}_3$, see above. Consequently the condition reduces to $\hat{K}_1 < \hat{K}_3$. So whenever the segment is non-empty, we will have $D'_K < 0$ at $\hat{K}_1$. From the total differential of $D$, we have $\frac{dD}{dK} = -\frac{D'_K}{D'_p}$, as the derivative of a function $p^o_w$, which thus is downward-sloping at $\hat{K}_1$.

Since the $p'_w$ function is linearly decreasing, and the $p^k_w$ function is linearly increasing in $K$, the two functions determine four domains in a $K$-$p_w$ diagram like Figure 3, with $\hat{K}_1$ being in the N-W domain where $p^o_w$ is decreasing in $K$. The $p^o_w$ function cannot hit the $p'_w$ function from this domain. So, eventually the $p^o_w$ function hits the $p^k_w$ function for a certain $K$, which we denote $\hat{K}_2$, where the function turns and becomes increasing in the N-E domain. The increasing part ends in $\hat{K}_3$. As in case a), as $p^o_w$ decreases it may become equal to $p_{w-1}$, and the two segments merge. The merged segment replaces segment $w$ in the proof.

**Case c):** $K - k_N < (a - c) < K + k_N \iff K_4 \leq K \leq K_5$: The Lagrange function becomes $L = (1 + \lambda_d) \sum_{s=1}^{\hat{N}} (p_s - c)(a_s - p_s)/b_s - \lambda_d (1 - \delta)(p_w - c)K - \lambda_d \delta(a - c - K(\hat{N} - 1)/\hat{N})^2\hat{N}/4$. We have $D^* = ((a - c)^2 + \Delta)/4 - (1 - \delta)((a_w - c)/2)K - \delta(a - c - K(\hat{N} - 1)/\hat{N})^2\hat{N}/4 \geq 0$. Recall that $K_4 = (a - c)\hat{N}/(\hat{N} + 1)$ and $K_5 = (a - c)\hat{N}/(\hat{N} - 1)$. We have $K_4 > K_3 \iff (a - c)\hat{N}/(\hat{N} + 1) > (a - c)/2\delta \iff \delta > (\hat{N} - 1)/2\hat{N}$, which is true by A3. From case b) we have $D > 0$ for any $K > K_3$,
and since \( \pi^n \) is a continuous function we will have \( D > 0 \) at \( K_4 \). From case a) we know that \( K_5 < \hat{K}_6 \), and thus \( D > 0 \) at \( K_5 \).

We have \( D > 0 \) at both ends of the interval. Furthermore we have \( \frac{\partial D^*/\partial K}{\partial K} = - (1 - \delta)(a_w - c)/2 + \delta \left( (a - c)(\hat{N} - 1) - K(\hat{N} - 1)^2/\hat{N} \right)/2 \) and \( \frac{\partial^2 D^*/\partial K^2}{\partial K^2} = - (\hat{N} - 1)^2/2\hat{N} < 0 \), which means that \( D^* \) is concave, which in turn means that \( D > 0 \) for case c).
Figure 1.
Figure 2.
Figure 3.
References.


