International Competition for R&D Investments*

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Abstract

Two jurisdictions compete to attract shares of the R&D investment budget of a large multinational enterprise, whose investments potentially confer positive spillovers on national firms. The firm has private information both about its efficiency and about spillovers. It is shown that strategic tax competition may lead to overinvestments relative to the first-best allocation, that the excessive investments occur in the country where the positive spillover effects are lowest, and that they are most severe for the least efficient firms. This occurs for sufficiently asymmetric spillovers, and implies that investments under competition are then excessively spread out and not properly concentrated to the country where spillovers would be largest.

1 Introduction

National governments compete to attract investments from multinational enterprises (MNEs), in particular those of R&D-intensive enterprises that may generate positive spillover effects (positive externalities) for national firms. Given the characteristics of MNEs1, e.g.}

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1MNEs tend to be important in industries and firms that are characterized by high levels of R&D relative to sales, a high value of intangible assets, a large share of professional and technical workers in their work force, products that are new or technically complex, and high levels of product differentiation and advertising (Markusen 1995).
technically complex products, there is every reason to believe that such a firm is better informed about its operations than are any of the national governments that it relates to. The international nature of an MNE and the high number of interfirm transactions also make it hard for tax authorities to observe true income and costs. Countries that try to attract investments from such a firm, must then take informational asymmetries into account in their policy design. Our aim in this paper is to construct a model that captures some of these typical aspects of multinationals, with a particular focus on spillover effects, and then study what implications tax competition has for investment allocations in this setting. Of particular interest is the issue of whether competition leads to misallocations in the sense that investments become too much spread out, and not properly concentrated in the country where they would yield the largest spillovers. The model shows (for parametric specifications) that competition does have this effect when spillovers are sufficiently asymmetric between countries. Comparing non-cooperative and cooperative investments in such cases, we find that non-cooperative investments are either higher in both countries, or higher (respectively lower) than cooperative investments in the country where spillovers are low (respectively high). In such cases there is thus a tendency for competition to induce investments that are either overall too high, or skewed towards overinvestments in the country with low spillovers combined with underinvestments in the country with high spillovers.

We consider a multi-principal regulation model of an MNE. The MNE (the agent) allocates its real investment portfolio in two jurisdictions, and has an option of redirecting parts of the investments from one of the jurisdictions to the other. The firm is assumed to have private information about its net operating profits and its productivity in the two countries. It is also assumed to have superior information about the potential positive spillovers that its activity may generate. As a part of a tax bargaining strategy the firm may then have an incentive to misrepresent its earning potential and the extent of spillovers in each individual country. Also, having investment opportunities in several countries, the MNE may try to reduce tax payments in each country by an implicit threat of directing a larger fraction of its investment to the neighbouring country. It is shown, among other things, that strategic tax competition in this setting may lead to overinvestments relative to the first-best allocation, that the excessive investments occur in the country where the positive spillover effects are lowest, and that they are most severe for the least efficient firms.
After this introduction the model is presented in section 2, and then sections 3 and 4 consider cooperative and non-cooperative equilibria, respectively. Spillover effects are analysed in detail for parametric functions in section 5, related literature is discussed in section 6, and section 7 concludes.

2 The model

We model a tax bargaining situation between a unique, large MNE (agent) and two independent countries (principals). There is strategic tax competition between the two countries where the firm is located, which is captured by a common agency framework. The MNE invests $K_1$ in country 1 and $K_2$ in country 2, yielding global profits (before taxes) $\Pi(K_1, K_2, \theta)$, where $\theta$ is an efficiency parameter. Investments are substitutes: $\partial^2 \Pi / \partial K_1 \partial K_2 < 0$. There are various reasons for assuming substitutability. First, there may be interaction effects in terms of joint costs, i.e. global profits may be of the form

$$\Pi(K_1, K_2, \theta) = N_1(K_1, \theta) + N_2(K_2, \theta) - C(K_1 + K_2).$$

(1)

where $C(K_1 + K_2)$ denotes joint costs, and $N_i(K_i, \theta)$ denotes operating profits for the affiliate in country $i$. Convex costs, i.e. $C' > 0$, $C'' > 0$ imply economic interaction effects among the two affiliates; an increase in investments in one of the countries implies a higher marginal joint cost, and thus lower marginal (global) profits from investments in the other country. These joint costs may have different interpretations. First, $K = K_1 + K_2$ may represent scarce human capital, e.g. management resources or technical personnel, where we assume that the MNE faces convex recruitment and training costs. Second, $K$ may represent real investments, where $C(K)$ are management and monitoring costs of the MNE. Economic management and co-ordination often become more demanding as the scale of international operations increase, i.e. $C(K)$ is likely to be convex. Finally, instead of – or in addition to – interaction effects from joint costs, there may in the case of imperfect competition be interaction effects in terms of market power. For example, if the two affiliates sell their output on the same market (e.g. in a third country), their activities are substitutes: high investments (and output) in affiliate 1 reduce the price obtained

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2 Alternatively, the model can be interpreted as describing multi-principal regulation of an internationally mobile industry with a continuum of small firms with different efficiency types for investments in the two countries and with different extent of local spillovers.

3 In addition there may be sunk investments in both countries.
by affiliate 2. Last, but not the least, an important case of a market interaction effect arises when \( K_1 \) and \( K_2 \) are investments in R&D; the marginal payoff on R&D-activities of affiliate 1 are then often lower the higher is the R&D activity of affiliate 2, e.g. due to a race to develop a new product or process. In the appendix we present a more detailed specification of the gross profit function \( \Pi() \) for this case.\(^4\)

The countries compete to attract scarce real investments from the MNE, and design their respective tax systems with a view to this competitive situation. The affiliates of the MNE are separate and independent entities, which means that they are subsidiaries and thus taxed at source. Letting \( r_1 \) and \( r_2 \) denote, respectively, the taxes paid to the two countries, the post-tax global profits of the firm are given by

\[
\pi = \Pi - r_1 - r_2.
\] (2)

The firm has private information about \( \theta \). It is presumed that if the firm is efficient in one country it is also an efficient operator in the other country; for reasons of tractability we assume that the firm has the same efficiency in the two countries. It is common knowledge among the two governments that the efficiency types are distributed on \([\underline{\theta}, \bar{\theta}]\) according to the cumulative distribution function \( F(\theta) \), with density \( f(\theta) > 0 \), where \( \underline{\theta} \) denotes the least and \( \bar{\theta} \) the most efficient type. The probability distribution satisfies the monotone hazard rate condition.\(^5\) Efficient types have higher net profits than less efficient types, both on average and at the margin: \( \partial \Pi / \partial \theta > 0 \) and \( \partial^2 \Pi / \partial \theta \partial K_j > 0, j = 1, 2 \); where the latter inequality is a single crossing condition. The joint return function has sufficiently decreasing marginal returns on capital so that it is optimal for the firm to invest in both countries.

The firm’s investments contribute (potentially) some positive spillovers/externalities \( \tilde{E}_j(K_j, \theta) \) to each country. The magnitudes of these externalities are known by the firm, but not by the authorities. These are also assumed to be positively correlated with the firm’s productivity; so \( \partial \tilde{E}_j(K_j, \theta) / \partial \theta > 0 \).

The MNE and the governments are risk neutral. For all efficiency types the affiliate’s net operating profits in each country are sufficiently high so that both governments always want to induce the domestic affiliate to make some investments in their home country.

\(^4\)Olsen (1993) analyses single-principal regulation of independent R&D units, and emphasizes the role of research activities as substitutes.

\(^5\)This condition is satisfied by most usual probability distributions, e.g., the normal, uniform, logistic and exponential distributions.
Domestic consumer surpluses in the two countries are unaffected by changes in the MNE’s production level, since the firm is assumed to be a price taker (or its market is outside the two countries). The governments have utilitarian objective functions: the social domestic welfare generated by an MNE of efficiency type $\theta$ is given by a weighted sum of the domestic taxes paid by the firm, the positive spillovers that it generates (for other domestic firms), and the MNE’s global profits:

$$W_j = (1 + \lambda_j) r_j + \tilde{E}_j(K_j, \theta) + \alpha_j \pi, \quad j = 1, 2,$$

where $\lambda_j$ is the general equilibrium shadow cost of public funds in country $j$, and $\alpha_j$ is the owner share of country $j$ in the MNE. We have that $\lambda_j > 0, \ j = 1, 2$, since marginal public expenditure is financed by distortive taxes. By inserting for Eq.(2), the social welfare function for country 1 can be restated as

$$W_1 = (1 + \lambda_1) (\Pi(K_1, K_2, \theta) - r_2 + E_1(K_1, \theta)) - (1 + \lambda_1 - \alpha_1) \pi,$$

where $E_1(K_j, \theta) = \frac{1}{1+\lambda_1} \tilde{E}_j(K_j, \theta)$. The social welfare consists of two terms. The first term is domestic social welfare under symmetric information, i.e. the welfare we would get if the government were able to tax away all the residual income of the MNE. The government’s revenue is in this case given by the MNE’s net operating profits minus foreign source tax, plus the ”adjusted” value of spillovers, and is multiplied by $(1 + \lambda_1)$ to obtain a welfare measure. The second term of the welfare function corrects for the loss of social welfare that stems from private information, i.e. the welfare loss to the country caused by the MNE keeping parts of the rent. The loss caused by imperfect rent extraction is equal to the MNE’s global rent multiplied by the difference between the welfare weights for income accruing to the MNE and the national government. The social welfare function for country 2 is analogous. Assuming that $\lambda_1 = \lambda_2 = \lambda$,

$$W = (1 + \lambda) (\Pi(K_1, K_2, \theta) + E_1(K_1, \theta) + E_2(K_2, \theta)) - (1 + \lambda - \alpha_1 - \alpha_2) \pi$$

is the cooperative welfare function.

Inserting $\pi(\theta) = 0$ in (4) and maximising with respect to $K_1$ and $K_2$, we obtain the first-best global allocation, given by $\frac{\partial \Pi}{\partial K_1} + \frac{\partial E_1}{\partial K_1} = \frac{\partial \Pi}{\partial K_2} + \frac{\partial E_2}{\partial K_2} = 0$. This allocation is

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6 The shadow cost of public funds is taken as exogenously given in our partial analysis. It can be endogenised by a general equilibrium model, without affecting the qualitative results. See Laffont and Tirole (1993), chapter 4.
obtained in the case of cooperating principals and symmetric information. The solution can be attained by imposing type-dependent taxes that correct for the externalities and at the same time extract the firm’s rents.

3 The second-best cooperative equilibrium

When the agent possesses private information and the principals cooperate, they seek to maximise the joint welfare given by Eq. (4), subject to incentive and participation constraints. The standard procedure for this case is (analogously to the single-principal case) to analyse it in terms of direct revelation mechanisms. The firm is then asked to make a report \( \hat{\theta} \), in response to which it is asked to invest \( K_1(\hat{\theta}) \) and \( K_2(\hat{\theta}) \) and to pay taxes \( r_1(\hat{\theta}) \) and \( r_2(\hat{\theta}) \). This yields profits \( \pi(\hat{\theta}, \theta) = \Pi(K_1(\hat{\theta}), K_2(\hat{\theta}), \theta) - r_1(\hat{\theta}) - r_2(\hat{\theta}) \). Incentive compatibility implies that the firm’s optimal choice of \( \hat{\theta} \) is \( \theta \), so it requires that

\[
\pi'(\theta) = \frac{\partial \Pi(K_1(\theta), K_2(\theta), \theta)}{\partial \theta}, \tag{5}
\]

where \( \pi(\theta) \equiv \pi(\theta, \theta) \). It can be shown that sufficient conditions for incentive compatibility are that \( dK_j(\theta)/d\theta \geq 0, j = 1, 2 \). We see from Eq. (5) that the firm’s rent is increasing in \( \theta \), i.e. to be willing to reveal their true types efficient types must be rewarded with a higher rent than inefficient types (downward adjacent incentive constraints). The optimal investment portfolio is characterised by the following system of equations:

\[
\frac{\partial E_j(K_1, \theta)}{\partial K_j} + \frac{\partial \Pi(K_1, K_2, \theta)}{\partial K_j} = \frac{1 + \lambda - \alpha_1 - \alpha_2}{1 + \lambda} \frac{\partial^2 \Pi(K_1, K_2, \theta)}{\partial \theta \partial K_j} \frac{1 - F(\theta)}{f(\theta)}, \quad j = 1, 2. \tag{6}
\]

Compared with the first-best global optimum, the presence of asymmetric information generates the additional right hand sides of (6), which represent marginal information costs. The investment portfolios are distorted in order to enhance the governments’ rent extraction from the MNE, which enables the government to reduce distortive taxes elsewhere in the economy. The distortions entail reductions of investment levels in both countries for all types except the most efficient one. At the same time, distortions of the capital allocation decision of the firm implies that the tax base is reduced. The investment portfolios are

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7 The Revelation Principle (see, e.g., Laffont and Tirole 1993) guarantees that, among all contracts the principal may use (including non-linear tax schedules), the best can always be achieved by the use of direct revelation mechanisms.

distorted to the point where the marginal deadweight loss equals the marginal deadweight losses in other sectors of the economy.

The optimal solution in (6) can, under relatively mild conditions, be implemented by a tax schedule \( R(K_1, K_2) \) where total tax payments depend only on realized investments. (See e.g. Laffont and Tirole (1986, 1993)).\(^9\) As a second-best response to asymmetric information, the optimal policy deviates from a level playing field (tax neutrality). A basic insight from regulation theory - applied to a tax setting - is to tailor corporate tax payments to firms’ characteristics. The flexibility imposed by differential tax enforcement enables the governments to raise welfare, relative to what is obtained with a common proportional effective tax rate. The induced investment distortions improve the governments’ ability to capture rent (by reducing the firm’s incentives to exploit its information advantage).

4 The second-best non-cooperative equilibrium

Consider now the case where the governments of the two countries do not cooperate, but rather compete to attract the firm’s investments. In this case the MNE relates to each government separately. The governments cannot credibly share information and they act non-cooperatively, i.e. we seek perfect Bayesian Nash Equilibria. We will focus on the regulatory problem of country 1; the decision problem of country 2 is analogous. Country 1 seeks to maximise expected domestic welfare, subject to incentive compatibility constraints and participation constraints for the firm; given the strategy of country 2. In this setting it is natural to assume that each country offers a tax schedule, specifying the firm’s tax obligation to that country as function of its investments there. So we are looking for an equilibrium in tax schedules (or menus) \( R_1(K_1), R_2(K_2) \).\(^{10}\) To interpret these tax schedules, we can envision the countries keeping the statutory corporate income tax rates fixed, and offering non-linear depreciation schedules and tax exemptions to attract investments from the MNE. In designing a non-linear corporate income tax scheme for

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\(^{9}\)In general, this will require a tax function that is not additive separable. To implement the cooperative solution taxes must thus in general be coordinated such that the marginal tax in one country depends on investments in both countries.

\(^{10}\)The Revelation Principle (in its usual form) is not generally valid for common agency, so there may exist equilibria in more general message spaces that can not be replicated as an equilibrium in direct revelation mechanisms (Martimort and Stole 1997, Peters 1999). The restriction to tax schedules seems reasonable in the current application.
internationally mobile firms, country 1 takes the depreciation schedules of country 2 as given. However, country 1 must take into account that its choice of strategy (tax schedule) may cause investment externalities: a change in the contract of country 1 may affect the agent’s investment in country 2, i.e. the agent’s choice of $K_1$ may affect its investment $K_2$, and thereby make it deviate from the investment level intended for it by country 2. Similar considerations apply for the latter country. A constraint on equilibrium contracts is imposed by the following feasible strategies: to capture a larger fraction of the MNE’s rent, each country will attempt to induce the MNE to deviate from the investment level intended for it by the other country. In equilibrium, however, all contracts are incentive compatible.

4.1 Equilibrium investments and taxes.

Given the tax function offered by country 2, we can use the Revelation Principle to find the optimal response from country 1. The MNE’s profits as a function of its report $\hat{\theta}_1$ to country 1 and its true type are now given by $\pi(\hat{\theta}_1, \theta) = \Pi(K_1(\hat{\theta}_1), K_2(\hat{\theta}_1), \theta) - R_1(K_1(\hat{\theta}_1)) - R_2(K_2(\hat{\theta}_1))$, where the firm’s investments in country 2 must satisfy $K_2(\hat{\theta}_1) = \arg \max_{K_2}[\Pi(K_1(\hat{\theta}_1), K_2, \theta) - R_2(K_2)]$. Incentive compatibility requires that the firm reports truthfully ($\hat{\theta}_1 = \theta$), which implies that (5) must hold for equilibrium profits in this non-cooperative case as well. In order for the firm’s investments in country 2 to be incentive compatible, we must further have

$$\frac{\partial \Pi}{\partial K_2}(K_1(\theta), K_2(\theta), \theta) - R'_2(K_2(\theta)) = 0.$$  \hspace{1cm} (7)

The decision problem of country 1 can now be seen as maximizing domestic welfare subject to the constraints (5), (7) and $\pi(\theta) \geq 0$.\textsuperscript{12} That is, the regulatory problem is similar to the cooperative case, with an additional restriction. Following a procedure similar to Martimort (1992, 1996) - see the Appendix - one can see that, if the system of differential equations below defines a pair of nondecreasing investment schedules \{$K_1(\theta), K_2(\theta)$\}, and

\textsuperscript{11}For a given tax function offered by country 2, it is not restrictive to consider only direct truth-telling mechanisms in country 1’s best response problem; the Revelation Principle holds for this single-principal problem. Restricting both principals simultaneously to such mechanisms, however, does in general affect the equilibrium.

\textsuperscript{12}The constraint (5) is only the first-order condition for the firm’s optimal choice of report. Under additional conditions (common implementability conditions) one can check ex post that the first-order condition is sufficient for optimality.
those schedules in addition satisfy a set of implementability conditions, they constitute a pure-strategy differentiable Nash equilibrium outcome for the common agency game:

$$\frac{\partial E_j}{\partial K_j} + \frac{\partial \Pi}{\partial K_j} = \frac{1 + \lambda - \alpha_j}{1 + \lambda} \left[ \frac{\partial^2 \Pi}{\partial \theta \partial K_j} + \frac{\partial^2 \Pi}{\partial \theta \partial K_i} \frac{\partial^2 \Pi}{\partial K_j, K_i} K_i'(\theta) \right] \left[ \frac{1 - F(\theta)}{f(\theta)} \right]. \quad (8)$$

To interpret and understand (8), we present here a heuristic derivation of this equilibrium condition. Country 1 takes the other country’s tax schedule as given, and recognises that the firm chooses foreign investments $K_2$, conditional on domestic investments $K_1$ in accordance with (7). A change in domestic taxes that induces a change $dK_1$ in domestic investments will then induce a change in foreign investments given by $dK_2 = -\Pi_{12} \Pi_{22} - \Pi_{00}$,

$$dK_2 dK_1 = -\Pi_{12} \Pi_{22} - \Pi_{00},$$

where $\Pi_{ij}$ denote second-order partials. A firm of type $\theta$ can, by mimicking a less efficient type $\theta - d\theta$, obtain profits $d\pi = \frac{\partial \Pi}{\partial \theta} d\theta$, so country 1 must design its tax scheme to allow for such profits (rents). A tax change that induces an investment change $dK_1$ for type $\theta$ will affect this type’s rents by

$$d\pi = \left[ \frac{\partial^2 \Pi}{\partial \theta \partial K_1} + \frac{\partial^2 \Pi}{\partial \theta \partial K_2} \frac{dK_2}{dK_1} \right] dK_1 d\theta \quad (9)$$

To preserve incentive compatibility, the same rent differential must be given to all better types, i.e. to a fraction $1 - F(\theta)$ of all types. The cost of this rent to country 1 is $(1 + \lambda - \alpha_1)d\pi$. This cost must be weighted against the efficiency gain, which from (3,7) is given by $(1 + \lambda)(\frac{\partial E_1}{\partial K_1} + \frac{\partial \Pi}{\partial K_1})dK_1 f(\theta) d\theta$. It follows that (8) expresses an optimal trade-off for country 1 (for $j = 1, i = 2$), provided that the last term in the square brackets is an adequate representation of the investment response $\frac{dK_2}{dK_1} = -\Pi_{12}/(\Pi_{22} - \Pi_{00})$. To verify that this is the case, we note that (7) must hold for all types in equilibrium, and by differentiation of this relation with respect to $\theta$ we see that $\frac{\Pi_{12}}{\Pi_{22} - \Pi_{00}} = \frac{\Pi_{12} K_i'}{\Pi_{20} + \Pi_{12} K_i'}$ holds in in equilibrium. Hence it follows that (8) is a pair of equilibrium conditions for the investment allocations that follow from the two countries’ strategic choice of tax schedules.

To characterize the equilibrium tax functions, note that (7) holds for each type $\theta$. Let $\theta_j(K_j)$ be the type that invests $K_j$ in country $j$ in equilibrium; i.e. $\theta_j()$ is the inverse of the equilibrium investment schedule $K_j(\theta)$; assuming that the latter is invertible (e.g.

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13 We get the result, traditional for common agency models, that the total tax payments of the MNE are uniquely determined in equilibrium, but not the distribution between the two countries. The latter would be determined outside the model by a bargaining game between the two governments, and will not affect the results of the model.
strictly increasing). Substituting $\theta = \theta_j(K_j)$ in (7) then implies

$$R'_j(K_j) = \frac{\partial \Pi}{\partial K_j}(K_j, K_i(\theta_j(K_j)), \theta_j(K_j))$$

(10)

This relation determines marginal tax rates for all investment levels that can be realized in equilibrium. Note that the tax schedule for an individual country must, for a given tax system in the other country, balance two concerns: (i) to induce production efficiency and (ii) to extract rents from the firm. The first calls for negative tax rates (subsidies) to correct for external spillovers from the firm, the second for positive tax rates to reduce investments and thereby facilitate rent extraction by relaxing the incentive constraints.

The equilibrium conditions (8) show that there are no distortions from the first-best ‘at the top’ (for type $\bar{\theta}$). It follows that the marginal tax rates given by (10) are negative for investments near $\bar{K}_j = K_j(\bar{\theta})$; in fact we have $R'_j(\bar{K}_j) = \frac{\partial \Pi}{\partial K_j}(\bar{K}_j, \bar{K}_i, \bar{\theta}) = -\frac{\partial E_j}{\partial K_j}(\bar{K}_j, \bar{\theta})$.

These marginal subsidies induce an efficient firm – which by assumption also generates high spillovers – to increase its investments beyond what would otherwise have been privately optimal. The concern to induce production efficiency thus dominates for such ‘high’ investments. In order to extract rents from the firm, less efficient types should be induced to invest less, and marginal taxes may for that reason be positive for lower investment levels $K_j$.

4.2 Non-cooperative vs. cooperative allocations.

Comparing the cooperative and the non-cooperative investment solutions, given by Eqs. (6) and (8), respectively, we see that in the latter case the marginal information costs contain an additional term, which accounts for the interaction effect of common agency. The added term is negative, i.e. the effectiveness of investment distortions as a means for relaxing the MNE’s incentive constraint are lower when the countries compete than when they collude. This term represents a second-order rent effect which calls for increasing the investment levels for all firms but the most efficient one. It is due to the ability that government $j$ (via a strategic tax policy) has to affect the investment $K_i$ that the MNE makes in country $i$. By imposing marginal taxes that induce an increase in the MNE’s domestic investments, the government of country $j$ causes investments to fall in country $i$ (for substitutes we have that $\frac{dK_i}{dK_j} < 0$), which has the effect, ceteris paribus, of increasing

$^{14}$The numerator is positive and we know from the necessary second-order conditions for common implementability that the denominator is negative.
the tax revenue of country \(j\) (fiscal externality). We cannot generally determine which of the two effects that dominates, i.e. an overinvestment result is not precluded.\(^{15}\)

The terms in Eqs. (6) and (8) involving \(\lambda\) and \(\alpha_i\)’s account for the difference in welfare weights of income that accrue to the government and the MNE (normalised by the shadow cost of public funds), and tells us how strong a motive the governments have to capture the MNE’s rents. In the non-cooperative case, the motive of rent extraction is always the stronger \((1 + \lambda - \alpha_j > 1 + \lambda - \alpha_j - \alpha_i)\), since the governments in this case do not internalise the profits that accrue to the investors in the other country (equity externalities).

Analogous to Stole (1992) and others we get fiscal externalities; when designing a tax policy that captures a larger fraction of the MNE’s investment budget, country \(j\) does not take into account the loss of tax revenue in country \(i\). Failing to account for this strategic interaction, this calls for investments to exceed the investment level determined by the cooperative solution. Unlike Stole (1992), by assigning positive welfare weights to the agent’s rents, we also get equity externalities: the governments do not internalise the effect on the rents that accrue to shareholders in the other country.\(^{16}\) This leads to a more aggressive rent collection by the governments, which implies higher distortions (lower investments) than in the cooperative case. The latter (underinvestment) effect will dominate when both \(\lambda\) and the firm’s outside owner share \((1 - \alpha_1 - \alpha_2)\) are small, the former (overinvestment) effect may dominate otherwise, e.g. when spillovers are small and the investments \(K_1, K_2\) are close substitutes for the firm.\(^{17}\) The following proposition summarizes this discussion; in the next section we will study in more detail how spillovers affect allocations.

**Proposition 1** When two countries compete to attract a large share of an MNE’s in-

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\(^{15}\)If there is a firm type \(\hat{\theta}_j\) for which the first- and second-order rent effects are exactly offsetting (i.e., for which the bracketed expression in Eq. (8) is zero), there are two types for which we get investment neutrality; \(\bar{\theta}\) and \(\hat{\theta}_j\).

\(^{16}\)A similar type of externality is present in Martimort (1996), where multiple regulators have biased objectives favoring an interest group. The externality effects are different, however, since the decisions considered there are complements, while ours are substitutes.

\(^{17}\)For the case of zero spillovers this was shown in Olsen and Osmundsen (1999a). For perfect substitutes \((\Pi_{jj} = \Pi_{12})\) the first-best investments will then satisfy the non-cooperative equilibrium conditions, and equilibrium investments will thus exceed cooperative investment levels, which are lower than first-best when \(\lambda\) and/or the outside owner share are positive. When the latter parameters are zero and investments are not perfect substitutes, the cooperative solution will coincide with the first-best optimum, while the non-cooperative equilibrium will entail underinvestments due to equity externalities.
vestment budget, and when there is private information about net operating profits and efficiency, the distortion of the investment portfolio is in equilibrium determined by a trade-off between a first-order (conventional) and a second-order (strategic interaction) rent effect. Compared with the cooperative solution, strategic interaction among governments introduces fiscal externalities and equity externalities, having opposite effects on investment levels. Hence, it cannot be generally determined whether tax competition leads to higher or lower investments.

We finally comment here the assumption maintained in this paper that the firm cannot completely escape taxation in one country by moving all of its operations to the other country. To understand some of the ramifications of this assumption, suppose instead that such moves are feasible for the firm. This imposes additional participation constraints on the principals; the firm’s (equilibrium) rent must then be at least as large as what the firm can obtain by escaping any one country. It appears that (at least for some functional forms) these additional constraints will only affect the distribution of rents between the parties: except for lump-sum rent transfers, the equilibrium allocation will be the same as before.\footnote{Technically, the new participation constraint facing principal $j$ will apparently bind only for the least efficient type: if $a(\theta)$ is the alternative rent the firm can obtain by escaping country $j$, and $\pi(\theta)$ is the rent in the ‘no-escape’ equilibrium, one can see that, at least for the quadratic-uniform case, that we have $\pi(\theta) \geq a'(\theta)$. Calzolari (1998) also makes this point.} This means that the countries cannot, when they compete, completely tax away all rents from the least efficient type of firm; even this type will keep some (mobility) rents. In this case, therefore, types with low efficiency will in general be better off under tax competition compared to tax coordination. More efficient types apparently may or may not be better off in the competitive regime; this seems to depend on how rapidly information rents increase with type.\footnote{This observation helps us understand the apparent ‘discontinuity’ between some full-information (e.g., Zodrow and Mieszkowski 1986) and our asymmetric-information model of tax competition: given the escape option it may depend on the extent of uncertainty whether tax competition is advantageous or not for the firm.}

5 A parametric case

By assuming specific functions, explicit regulatory mechanisms may be derived. We solve for the case of quadratic/linear functions and a uniform distribution. We further assume
here that the firm’s private investment returns are symmetric between the two countries. The specific functional forms are:

\[ E_j(K_j, \theta) = e_j \theta K_j \]

\[ \Pi(K_1, K_2, \theta) = (m \theta + k) \sum_j K_j - \frac{1}{2} q \sum_j K_j^2 - \frac{1}{2} a (K_1 + K_2)^2, \]

with \( m, q, a > 0 \); and

\[ f(\theta) = 1, \quad \theta \in [0, 1]. \]

For the first-best allocation we get two equations:

\[ \frac{\partial \Pi}{\partial K_j} + \frac{\partial E_j}{\partial K_j} = 0, \quad j = 1, 2, \]

which yield the solutions

\[ K_j^F(\theta) = \frac{(m + e_j) (q + a) - (m + e_i) a}{(q + a)^2 - a^2} \theta + \frac{k q}{(q + a)^2 - a^2} \equiv K_j^F \cdot \theta + L_j^F, \]

where the identity defines the slope \((K_j^F)\) and intercept \((L_j^F)\) of this linear relation. We assume \((m + e_j) (q + a) - (m + e_i) a > 0\), \(i, j = 1, 2\), implying that the first-best investment levels are increasing in \( \theta \) in both countries.

The cooperative solution as given by Eq. (6), implies two equations of the form

\[(m + e_j) \theta + k - (q + a)K_j - aK_i = \frac{1 + \lambda - \alpha_1 - \alpha_2}{1 + \lambda} m (1 - \theta). \]

The cooperative equilibrium also yields investment levels that are linear in \( \theta \); specifically \( K_jC(\theta) = K_j^C \cdot \theta + L_j^C \), where \( K_j^C = K_j^F + \gamma \frac{ma}{(q + a)^2 - a^2}, \quad \gamma = \frac{1 + \lambda - \alpha_1 - \alpha_2}{1 + \lambda} \), and \( L_j^C \) is such that there is no distortion from the first-best allocation for the best type \( (\theta = 1) \), i.e. \( L_j^C = -K_j^C + K_j^F + L_j^F \). There is underinvestment in country \( j \) if the cooperative investment schedule in that country is steeper than the corresponding first-best schedule (i.e. if \( K_j^C > K_j^F \)). We see that under our assumptions this will certainly be the case for both countries as long as \( \gamma > 0 \), i.e. as long as \( \lambda > 0 \) and/or \( \alpha_1 + \alpha_2 < 1 \). So we may state:

**Proposition 2** In the uniform-quadratic case where the countries are symmetric except possibly for spillover effects, the cooperative solution yields underinvestment (relative to first-best) in both countries. The distortions increase with increasing \( \lambda \) and/or increasing outside ownership \((1 - \alpha_1 - \alpha_2)\).

### 5.1 Common-agency equilibrium

The equations that define the common-agency equilibrium take in this example the form:

\[(m + e_j) \theta + k - (q + a)K_j - aK_i = \frac{1 + \lambda - \alpha_j}{1 + \lambda} \left[ m + \frac{aK_i^j(\theta)}{aK_i^j(\theta) - m} \right] (1 - \theta), \quad j = 1, 2. \]

Seeking linear solutions \( K_j(\theta) = L_j + K_j^* \cdot \theta, \quad j = 1, 2 \), to these differential equations, we find that the slope parameters \( (K_j^*) \) must satisfy

\[(m + e_j) - (q + a)K_j^* - aK_i^* = \frac{1 + \lambda - \alpha_j}{1 + \lambda} \left[ m + \frac{maK_j^*}{aK_j^* - m} \right], \quad j = 1, 2, \]

(11)
while the intercept parameters \((L_j)\) can be determined by the condition that there should be no distortions for the best type \((\theta = 1)\), i.e. by \(L_j + K'_j = L_jF + K'_jF\). In order to qualify as a common-agency equilibrium, the linear solutions must in addition be commonly implementable (see the appendix). Necessary and sufficient conditions for common implementability in this context are \(mK'_i \geq aK'_1K'_2\), \(i = 1, 2\) and \(K'_iK'_2 \left(1 - \frac{\epsilon}{m} \left[K'_1 + K'_2\right]\right) \geq 0\). The following conditions are therefore sufficient

\[
0 \leq \frac{a}{m} \frac{K'_j}{K'_i} \leq 1, \quad j = 1, 2 \quad \text{and} \quad \frac{a}{m} K'_1 + \frac{a}{m} K'_2 \leq 1. \quad (12)
\]

The equilibrium tax schedules can now be derived from (10). For this purpose it is convenient to write \(K_j(\theta) = \bar{K}_j + K'_j \cdot (1 - \theta)\), where \(\bar{K}_j\) is the first-best investment for the most efficient type. A little algebra then yields (see the appendix)

\[
R'_j(K_j) = \left(-\frac{m - aK'_i}{K'_j} + (q + a)\right) (\bar{K}_j - K_j) - e_j. \quad (13)
\]

The expression in the first parenthesis is positive, implying that marginal taxes are decreasing with increasing investments in equilibrium. The expression in the first parenthesis is equal to \(R''_j\), and by identifying terms we see that the relation \(\frac{-\Pi_{12}}{\Pi_{ij} - \bar{K}_j} = \frac{\Pi_{12}K'_1}{\Pi_{ij} + \Pi_{12}K'_i}\), where either side captures the strategic effect \(\frac{dK_j}{dK_i}\), holds in equilibrium. The formula further confirms that the marginal investment tax is negative and equal in magnitude to the marginal spillover effect (for the best type) when \(K_j = \bar{K}_j\). Moreover, for low investments we see that the marginal tax rate is positive; for \(K_j = K_j(0) = \bar{K}_j - K'_j\) we have \(R'_j(K_j) = -m + aK'_1 + (q + a)K'_j - e_j\), and the last expression is positive according to (11).

### 5.2 The fully symmetric case

We solve now for the fully symmetric case of quadratic/linear functions and a uniform distribution: In addition to the assumptions above we then assume symmetric externality effects \((e_1 = e_2 = e)\) and symmetric owner shares: \(\alpha_1 = \alpha_2\).

The first-best allocation is then symmetric between countries and given by \(K_{jF}(\theta) = \frac{m}{a} \left(\frac{1 + \epsilon}{Q + 1}\right) \theta + \frac{k}{Q + 1} \equiv K'_{jF} \cdot \theta + L_{jF}\), where \(\epsilon = \frac{e}{m}\) and \(Q = \frac{a}{2} + 1\). Here \(\epsilon\) is a measure of the relative strength of the spillovers from the firm. We see that investments are higher, the stronger are these spillovers. But note that for the firm of type \(\theta = 0\) there are no spillover effects, and for this type investments are not affected. The first-best investment profile is illustrated by the heavy line in Figure 1.
The cooperative solution also yields investment levels that are symmetric and linear in $\theta$; specifically $K_j^C(\theta) = K_j^F \cdot \theta + L_j^C$, where $K_j^C = K_j^F + \frac{m \gamma}{a Q+1}$, $\gamma = \frac{1+\lambda-2\alpha_1}{1+\lambda}$, and $L_j^C$ is such that there is no distortion from the first-best allocation for the best type, i.e. $L_j^C = -K_j^C + K_j^F + L_j^F$. We know from the previous section that this solution yields underinvestment in both countries. This is illustrated in Figure 1, where the solid line depicts the symmetric information equilibrium (F-schedule), and the thin line depicts the case with asymmetric information and colluding principals (C-schedule).

For the non-cooperative case (common agency), the linear solutions are given by $K_j(\theta) = K_j^F \cdot \theta + L_j$, $j = 1, 2$, where the (symmetric) slopes now can be found explicitly by solving equations (11). We find 

$$K_j(\theta) = \frac{m}{a} \left( \frac{1 + \varepsilon + \frac{k}{m}}{Q+1} - (1 - \theta)x \right) \equiv K_j' \cdot \theta + L_j,$$

where

$$x = \frac{1}{2(1+Q)} \left( \varepsilon + 2 + Q + 2\gamma_1 - \sqrt{\varepsilon^2 - 2Q\varepsilon + 4\varepsilon\gamma_1 + 4\gamma_1 + Q^2 + 4\gamma_1^2} \right)$$

and $\gamma_1 = 1 - \frac{\alpha}{2a}$. In order that the solution be commonly implementable, it is necessary and sufficient that $x = \frac{m}{a} K_j'$ satisfies $0 \leq 2x \leq 1$ (see (12) above). Straightforward algebra shows that this condition amounts to $2\varepsilon \leq \frac{2a}{m}$.

The equilibrium tax schedules are from (13) symmetric and given by

$$R'(K_j) = (-\frac{m}{K_j'} + q + 2a)(K_j - K_j) - e$$

where $\frac{m}{K_j'} = \frac{m}{x}$. Note that when $2\varepsilon \rightarrow \frac{2a}{m}$ (and thus $\varepsilon = \frac{\varepsilon}{m} \rightarrow \frac{1}{2}(Q - 1)$) we get $x \rightarrow \frac{1}{2}$ and thus $\frac{m}{K_j'} = \frac{2a}{2} \rightarrow 2a$, which implies $R''(K_j) \rightarrow -q = \Pi_{1j} - \Pi_{12}$. In this limiting case (when the spillover gets to be sufficiently strong) we thus see that the strategic investment effects in equilibrium become equal to $-1$ in both countries; $dK_i dK_i = \frac{\Pi_{1j}}{\Pi_{1j} - R'} \rightarrow -1$. Neither country will then have an incentive to distort investments away from the first-best. There is nothing to be gained by such unilateral distortions, because any induced distortion $dK_i$ of domestic investments will be met by the firm by an offsetting change of foreign investments $(dK_j = -dK_i)$, and this will leave the firm’s rents unaltered, see (9). Any domestic tax modification that induces distortions away from the first-best will therefore only reduce allocative efficiency and not enable the country to capture any more of the firm’s rents. Equilibrium investments will therefore not be distorted away from the first-
best in this case. The tax functions that implement these investments are in the limiting case given by \( R'(K_j) = q(K_j - K_j) - e^0 \), where \( e^0 = \frac{mq}{2a} \) and \( K_j = \frac{m}{2a} + \frac{k}{q+2a} \).

Let us now compare the first-best and the common agency investment schedules for smaller spillover effects (\( 2e_m < \frac{q}{a} \)). There is underinvestment in common agency (relative to FB) if and only if the common agency-schedule is steeper than the first-best schedule, i.e. if and only if \( K'_j = \frac{m}{a}x > K'_F = \frac{m+e}{Q+1} \). This is equivalent to \((Q+1)x > 1 + \varepsilon\). Algebraic manipulations show that this condition is also equivalent to \( \frac{q}{a} > 2e_m \). Thus, the common agency equilibrium entails underinvestment relative to the first-best.

We next turn to a comparison of the common agency and the coordinated solutions. Using the explicit solutions derived above we can show (see the appendix):

**Proposition 3** In the symmetric uniform-quadratic case, both tax competition (common agency) and tax coordination (single agency) induce underinvestment relative to first-best investments. There is lower investment – and hence lower profits for the firm – under competition than under coordination, if and only if \( \frac{q}{a} > 2e_m \). Otherwise, e.g. for larger spillovers, a larger cost of public funds and/or a larger outside owner share (smaller \( \alpha_1 \)), there is higher investment under tax competition than under tax coordination.

As spillovers become sufficiently large (as \( \frac{e_m}{m} \to \frac{1}{2} \)), the competitive equilibrium investment levels become equal to first-best investments.

For given technology we see that there is underinvestment – and hence lower profits for the firm – under tax competition than under tax coordination when the domestic owner share (\( \alpha_1 \)) is high. This is in accordance with the discussion in section 4, since high domestic owner shares imply strong equity externalities, and this tends to yield lower investments under competition than under coordination.

Other things equal, smaller spillovers (\( e \)) will also generate underinvestment in the competitive tax regime (provided \( \frac{q}{a} > 2 \frac{e_m+\lambda}{\alpha_1} - 4 \)). Lower spillovers will reduce investments both under competition and under coordination, but more so in the former case. The

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20 To check that the equilibrium investment schedules in the limiting case are first-best, note that their slopes satisfy \( K'_j = \frac{m}{a}x \), which is the slope of the first-best schedule for \( 2e_m = \frac{q}{a} \).

21 Note that the symmetry assumptions imply \( \alpha_1 \leq \frac{1}{2} \) and thus \( \frac{1+\lambda}{\alpha_1} \geq 2 \). For \( \alpha_1 = \frac{1}{2} \) and \( \lambda = 0 \) the condition for underinvestment reduces to \( \frac{q}{a} \geq 2e_m \), which is the condition for implementability. Thus, for a feasible technology the condition for underinvestment in the proposition is satisfied if \( \alpha_1 \) is high and \( \lambda \) is low.
reason for this is that lower spillovers reduce the strategic effect, and as this effect becomes weaker, equilibrium investments are reduced.

For larger spillovers (so that \( \frac{a}{2} + 4 < \frac{1+\lambda}{\alpha_1} \left( \frac{c}{m} + 2 \right) \)) there will be overinvestment under tax competition compared to tax cooperation. Larger spillovers intensify the competition to attract valuable investments, and this leads to overinvestments relative to the cooperative solution. Both regimes yield investments below the first-best levels, but in the limit, as spillovers become sufficiently large (\( \frac{c}{m} \to \frac{1}{2} \)), the competitive equilibrium investments become equal to first-best investments. As we have seen, the latter occurs because in the limit the strategic effect \( \frac{dK_i}{dK_j} \) becomes equal to \(-1\); and for such a strong strategic effect there is in equilibrium no incentive for any country to distort investments away from the first best in order to capture rents.

5.3 The case of asymmetric spillovers

We have seen that under full symmetry both the cooperative and non-cooperative solutions generate underinvestment compared to the first-best, although to different extents. We consider now the case of asymmetric spillovers. (Otherwise the countries are assumed to be symmetric.) An interesting and from a policy viewpoint important issue is whether competition in such a case will lead to misallocations in the sense of investments becoming too much spread out, and not properly concentrated in the country where they would yield the largest spillovers. The model shows that competition does have this effect when spillovers are sufficiently asymmetric between countries. Comparing non-cooperative and cooperative investments in such cases, we find that non-cooperative investments are either higher in both countries, or higher (respectively lower) than cooperative investments in the country where spillovers are low (respectively high). In fact, not only are non-cooperative investments higher than cooperative (second-best) investments in the low-spillover country, the they are higher than even the first-best investment levels there. In the following we first derive and explain the latter result, and then turn to the comparison of non-cooperative

22 The strategic investment effect is in the symmetric case is given by \( \frac{dE_i}{dK_j} = aK_i' \), where the (common) slope \( K_i' \) decreases as \( e \) decreases. A lower \( e \) thus reduces the (absolute value of) the strategic effect.

23 It should be noted that it is the competition over rents that becomes intensified when the spillover parameter \( e \) increases. If spillovers were of the form \( E_j = e \theta K_j + f K_j \), then variations in the parameter \( f \) would not affect the intensity of competition, as measured by the strategic effect. An increase of \( f \) would not affect the strategic effect, and would thus lead to an equal increase of investments under competition and cooperation.
(common agency) and cooperative (single agency) outcomes.

To simplify the analysis, we shall assume that spillovers are non-existent in one country, here taken to be country 2. (This fact is common knowledge.) So we have \( e_1 > 0, e_2 = 0. \) The firm continues to have private knowledge about the extent of spillovers in the other country. In particular, it knows whether it does in fact generate any spillovers at all in that country, which is the case if and only if it is of type \( \theta > 0. \) (For type \( \theta = 0 \) there are no spillovers in either country, since then \( E_j(K_j, \theta) = e_j\theta K_j = 0. \)

**Non-cooperative (common agency) vs. first-best outcomes.** Using \( \varepsilon_1 = \frac{e_1}{m} \) and \( Q = \frac{a}{m} + 1, \) the first-best solution is now given by

\[
K_{1F}(\theta) = \frac{m}{a} \left( \frac{(1 + \varepsilon_1)Q - 1}{Q^2 - 1} \theta + \frac{k}{Q + 1} \right), \quad K_{2F}(\theta) = \frac{m}{a} \left( \frac{Q - (1 + \varepsilon_1)}{Q^2 - 1} \theta + \frac{k}{Q + 1} \right)
\]

We see that investments are higher in the country where spillovers are present, and lower in the other country. For a firm of type \( \theta = 0, \) which generates no spillovers in either country, the investments are equal across countries.

For the **non-cooperative case** (common agency), the linear solutions are given by \( K_j(\theta) = K'_j \cdot \theta + L_j, \ j = 1, 2, \) where the slopes satisfy equations (11). In the appendix we show the following result.

**Lemma.** For any \( e_1 \geq 0 \) such that \( \frac{m}{m} \leq \varepsilon_1^0 = \frac{(Q-1)Q}{Q^2+\gamma_1}, \) the following holds: Equations (11) yield unique solutions for the slopes \( K'_1, K'_2 \) that satisfy the implementability conditions (12). As \( e_1 \to \varepsilon_1^0m, \) we have \( K'_1 \to \frac{m}{a} \) and \( K'_2 \to 0. \)

From this lemma we can conclude that for a spillover parameter \( e_1 \) sufficiently close to \( m\varepsilon^0_1 \) (so that \( \varepsilon_1 \) is close to \( \varepsilon^0_1 \)), the common-agency investment schedules \( K_j(\theta) = K'_j \cdot \theta + L_j \) have slopes \( K'_1 \approx \frac{m}{a} \) in country 1 and \( K'_2 \approx 0 \) in country 2. We will now compare these slopes to the slopes of the corresponding first-best schedules.

The latter slopes are, for \( \varepsilon_1 = \varepsilon^0_1, \) given by \( K'_{1F} = \frac{m}{a} \frac{(1+\varepsilon^0_1)Q - 1}{Q^2 - 1} \) and \( K'_{2F} = \frac{m}{a} \frac{Q - (1+\varepsilon^0_1)}{Q^2 - 1}. \) Note that \( \varepsilon^0_1 < Q - 1, \) and hence \( (1+\varepsilon^0_1)Q < Q^2, \) which implies \( K'_{1F} < \frac{m}{a}. \) Also \( Q - (1+\varepsilon^0_1) = (Q-1)\frac{2\gamma_1}{Q^2+\gamma_1}, \) so \( K'_{2F} > \frac{m}{Q^2+\gamma_1} > 0. \) This shows that for \( \varepsilon_1 \) close to \( \varepsilon^0_1 \) we have \( K'_1 > K'_{1F} \) and \( K'_2 < K'_2F. \) Hence for spillovers such that \( e_1 \) is close to \( m\varepsilon^0_1, \) it is the case that the common-agency investment schedules are steeper in country 1 and flatter in country 2, compared to the first-best. Figure 2 illustrates this configuration. These considerations prove the following.
Proposition 4 There are cases where the common-agency solution entails overinvestment (relative to first-best) in the country where spillovers are known to be low, and underinvestment in the country where spillovers are (for every type of firm) known to be higher. This is in particular true in the uniform-quadratic case when (i) the countries are symmetric except for spillover effects, (ii) there are no spillovers in one country \((E_2(K_2, \theta) \equiv 0)\) and (iii) there are positive and sufficiently strong spillovers in the other country \((E_1(K_1, \theta) = e_1 \theta K_1\) with \(e_1\) close to \(m \varepsilon_0\).)

Some intuition for why an equilibrium can have these features can be developed as follows. Consider country 2, and suppose that taxation in country 1 has reduced investments there to a level below the first-best level \((K_1 < K_{1F})\). Since lower investments in country 1 increase marginal profits in country 2, it is then advantageous with respect to production efficiency for the latter country to increase investments beyond the first-best \((\frac{\partial \Pi}{\partial K_2}(K_1, K_{2F}, \theta) > 0 \text{ when } K_1 < K_{1F}(\theta))\). At the same time the social costs – in terms of increased rents accruing to the firm – associated with higher domestic investments are low, because the strategic investment effect is strong under the given conditions. (Recall that the strategic effect reduces the rents associated with a given increase in domestic investments.) The strategic effect is given by \(\frac{dK_1}{dK_2} = \frac{-a K_0}{m - a K_2}\), and this expression is large (in absolute value) under the given conditions because \(K_0\) and \(K_0\) are then 'large' and 'small', respectively. Overall, it is therefore advantageous for country 2 to increase domestic investments beyond the first-best level. Conversely, for country 1 the strategic effect \((\frac{dK_2}{dK_1})\) is small, and increased domestic investments are then costly due to the considerable rents that they induce. This country therefore imposes taxes that induce underinvestment domestically. To illustrate these effects further, we consider a specific example.

**Example.** Suppose \(q = a = m\) and \(\frac{a}{m} = \frac{1}{4}, e_2 = 0.\) Then (since \(Q = 2\) and \(\varepsilon_1 = \frac{1}{4}\)) the first-best investment schedules satisfy \(K'_{1F} = \frac{(1+\varepsilon_1)Q-1}{Q^2-1} = \frac{1}{2}\) and \(K'_{2F} = \frac{Q-(1+\varepsilon_1)}{Q^2-1} = \frac{1}{4}.\) Suppose further that \(\frac{a}{1+a} = \frac{1}{4}\), so that \(\gamma_1 = \frac{3}{4}.\) Then we can check (from the equilibrium conditions (11)) that the common-agency schedules satisfy \(K'_1 = \frac{1}{2}\) and \(K'_2 = \frac{1}{4}.\) The assumed parameter values thus yield \(K'_1 > K'_{1F}\) and \(K'_2 = K'_{2F},\) which tells us that the corresponding equilibrium entails underinvestment relative to the first-best in country 1, and exactly first-best investments in country 2. (We have here \(\varepsilon'^0_1 = \frac{8}{11} > \varepsilon_1;\) a stronger spillover effect in country 1 would have generated overinvestment in country 2.)

The associated equilibrium tax schedules can be obtained from (13), and we get
\[ R_j^*(K_j) = m(-\frac{1-K_j}{K_j} + 2)(\bar{K}_j - K_j) - e_j. \] Hence we have \( R_1^*(K_1) = m\frac{2}{3}(\bar{K}_1 - K_1) - \frac{1}{3}m \) and \( R_2^*(K_2) = m\frac{2}{3}(\bar{K}_2 - K_2) \), where \( \bar{K}_1 = \frac{1}{2} + \frac{1}{3}k \), \( \bar{K}_2 = \frac{1}{2} + \frac{1}{3}k \). In country 2, where there are no spillovers, the marginal tax rate is non-negative, decreasing and equal to zero for \( K_2 = \bar{K}_2 \). In country 1 the marginal tax rate is also decreasing, it is further positive for 'small' investments, but negative for 'large' ones. For \( K_1 = \bar{K}_1 \) the marginal subsidy in country 1 equals the marginal domestic spillover effect.

Given these tax schedules we can compute the strategic investment effects from \( dK_j dK_i = \frac{-\Pi_{ij}}{\Pi_{jj}} \), and we obtain \( \frac{dK_2}{dK_1} = \frac{1}{-2+1/2} = -\frac{2}{3} \), \( \frac{dK_1}{dK_2} = \frac{-1}{-2+4/5} = -\frac{5}{6} \). The latter, applying to country 2, is much stronger than the former, applying to country 1. Country 1 has therefore a much stronger incentive than country 2 to distort investments to capture rents. (By a marginal investment distortion, country 1 can capture rents amounting to \( m\frac{1}{3}dK_1 d\theta \), while country 2 by a similar domestic operation only can capture \( m\frac{1}{6}dK_2 d\theta \), see (9).) If country 2 imposed a lump-sum tax, then the tax schedule in country 1 would generate underinvestments (relative to the first-best) in that country, and – by a substitution effect on the part of the firm – overinvestments in country 2. But since there are no spillovers in country 2, the firm’s investments there would be conditionally efficient, given its investments in the other country. The government of country 2 has a relatively weak incentive to distort these investments to capture rents. The small distortion, achieved by imposing positive, but relatively small marginal taxes, reduces investments in country 2 somewhat, and – for the given parameters – to a level that coincides with what is unconditionally first-best in that country. Conversely, given country 2’s tax schedule, country 1 has a relatively strong incentive to distort domestic investments (from what would be conditionally efficient there), and this leads to investments that are lower than the unconditionally first-best levels in country 1.

The example considered a case of a ‘moderately strong’ spillover in country 1. As the spillover in that country becomes stronger, the tendency for equilibrium overinvestments to occur in country 2 also gets stronger. In the limit, as \( \varepsilon_1 \rightarrow \varepsilon_1^0 \), the strategic effect for country 2 \( \left( \frac{dK_1}{dK_2} \right) \) approaches \(-1\), while that for country 1 approaches a limit with absolute value strictly less than one. (That limit can, from the equilibrium conditions (11), be seen to be \(-\frac{1+\gamma_1}{Q \gamma_1+1} \).) In the limit country 2 has therefore no incentive to unilaterally distort domestic investments from what is conditionally efficient, given the firm’s investments in the other country. Since there are no spillovers in country 2, a zero marginal tax rate will
ensure such investments domestically.\textsuperscript{24} Country 1 on the other hand, does have an incentive to distort investments in order to capture rents, and hence we get underinvestments in that country. From (13) and Lemma 1 we see that country 1’s equilibrium tax schedule in the limit is given by $R'_1(K_1) = q(\bar{K}_1 - K_1) - m\varepsilon_1^0$.

**Non-cooperative (common agency) vs. cooperative (single agency) solutions.** We now turn to a comparison of the non-cooperative equilibrium with the cooperative solution. This analysis shows that competition does, for sufficiently asymmetric spillovers, lead to misallocations in the form of investments being either excessive in both countries, or excessive in the country where spillovers are lowest and at the same time insufficient in the country where spillovers are highest.

To see this, first note that the cooperative solution, due to the absence of strategic effects, induces underinvestments relative to the first-best in both countries (Proposition 2). Since the non-cooperative outcome yields overinvestments relative to the first-best in one country when spillovers are sufficiently asymmetric (Proposition 4), it follows immediately that these investments are also higher than investments under cooperation in that country. Thus, for sufficiently asymmetric spillovers competition leads to too much investments in the country where spillovers are low (here zero, in fact). Comparing non-cooperative and cooperative investments in the other, high-spillover country for such cases, we find that their relative magnitudes depend on the parameters of the model. For some set of parameters, notably when domestic owner shares are large and/or the cost of public funds is small, the non-cooperative investments are (for highly asymmetric spillovers) higher than the cooperative ones in both countries. For the complementary parameter set the non-cooperative investments are higher than the cooperative ones in the low-spillover country, and at the same time lower than the cooperative ones in the other, high-spillover country. More precisely:

**Proposition 5** For the uniform-quadratic case the following holds. If the countries have sufficiently asymmetric spillovers ($E_2(K_2, \theta) \equiv 0$ and $E_1(K_1, \theta) = c_1 \theta K_1$ with $c_1$ close to $m\varepsilon_1^0$) then non-cooperative investments are higher than cooperative investments in the country where spillovers are low, while the corresponding investments in the other, high-spillover country may, depending on the parameters, be lower or higher than the cooperative investments.

\textsuperscript{24}From (13) and (11), we find that in the limit (as $\varepsilon_1 \rightarrow \varepsilon_1^0$) country 2’s tax schedule satisfies $R'_2 \rightarrow q\gamma_2$. The marginal tax rate is nevertheless effectively zero, since the formula (13) applies only on a vanishingly small interval; $\bar{K}_2 - K_2 \rightarrow 0$.  

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investments there. They are lower if and only if \( \frac{a}{\alpha} > \frac{1 + \lambda}{\alpha}\left(2\left(1 - \frac{\alpha}{1 + \lambda}\right)^2 - 1\right) \), which holds e.g. if the domestic owner shares are large and/or the cost of public funds is small.

6 Related Literature

In our model, competition among the countries to attract potentially valuable investments thus results in excessive amount of investments being made in one country, and insufficient amounts being made in the other. This result is in contrast with tax competition models with symmetric information, in which the competition for attracting real investments invariably causes source taxes to fall and investments to rise, see, e.g. Zodrow and Mieszkowski (1986).

The focus of the present model is on private information about spillover effects and productivity, i.e. we do not specifically address the issues of intra-firm trade and transfer pricing. For an analysis of transfer pricing regulation see, e.g. Bond and Gresik (1996).25 In that article the competing governments control complementary activities, whereas in our model the relevant activities are substitutes. Our economic focus is different, and by addressing the issues of spillover effects and externalities, our analytical perspective is different. We also get different qualitative results. Bond and Gresik find that under asymmetric information the firm’s activity level and information rents always are lower when the principals compete than when they cooperate, and that the activity level always is highest in the first-best case. We find that the activity level (investments) and rents under competition may be either higher or lower than under cooperation. We also show that the activity level under common agency may even exceed the first-best level in one country.

Our model is in some respects an extension of Osmundsen, Hagen and Schjelderup (1998); a partial model where a single principal regulates a continuum of mobile firms that have private information about their mobility costs. That analysis presumes a passive foreign government, which may be unrealistic since it implies a transfer of tax revenue from the foreign country to the home country. We extend the model to take into account strategic interaction between the governments, and by accounting for externalities.

Our model is also related to Laussel and Lebreton (1995), who analyse taxation of a

25A home and a host country use trade taxes to regulate an MNE which has private information about the cost of an intermediate good that is sold from the parent to a subsidiary in the host country.
large investor which possesses an exogenous amount of capital that it may allocate in two locations.\textsuperscript{26} We extend this analysis by allowing for spillover effects and national ownership, which affects the qualitative results by introducing equity externalities. Moreover, in our model the level of capital is endogenous, and whereas the firm has private information about the amount of capital it possesses in Laussel and Lebreton, we focus on private information about spillover effects and efficiency.\textsuperscript{27}

A different, yet related multiprincipal regulatory problem is analysed by Mezzetti (1997), who considers a case where an agent has private information about his relative productivity in the tasks he performs for two principals. In contrast, our focus is on private information about the absolute efficiency level. Also, we introduce spillover effects. Another difference is that we address a case of substitutes, whereas in Mezzetti’s model there is complementarity between the the agent’s tasks. The results also differ in important ways. Whereas Mezzetti finds that the agent’s information rent is always higher under tax competition than under cooperation, we find that the rent may be lower in some cases. Moreover, Mezzetti’s model exhibits countervailing incentives; this is not the case in our model.\textsuperscript{28}

The present paper extends Olsen and Osmundsen (1999a) by introducing externalities. Whereas the latter paper focuses on the effects of the national distribution of ownership in the firm, the present paper addresses the impacts of (possibly asymmetric) spillover effects on tax equilibria and the firm’s real investment portfolio.

\textsuperscript{26}A similar setup is found in Haaparanta (1996), but under perfect information. Haaparanta analyses a subsidy game where two governments, maximising the net wage income, compete to attract investments of a single firm.

\textsuperscript{27}Our model is also somewhat related to Biglaiser and Mezzetti (1993), in which two principals compete for the exclusive services of an agent that has private information about his or her effort and productivity. Whereas we focus on a multinational enterprise that divides its activities between several countries, Biglaiser and Mezzetti analyse a case where a worker must work full time for a single company.

\textsuperscript{28}In Olsen and Osmundsen (1999b) we extend the present model (in absence of spillover effects) to include an outside option, and this option is also seen to generate countervailing incentives; i.e. incentive constraints bind ‘upwards’ as well as ‘downwards’ on the type interval. The different information structures in the two models have quite different implications, though; whereas Mezzetti obtains equilibria that are unique and exhibit pooling for a range of intermediate types, we obtain non-unique and fully separating equilibria.
7 Conclusion

With enhanced international mobility of the corporate tax base, tax competition is reinforced and national governments experience more problems in raising revenue. Foreign direct investments have been rapidly increasing,\(^{29}\) and recent empirical research show that effective tax rates are important factors for determining the localisation decisions of multinational enterprises (MNEs).\(^{30}\) We have considered a situation where two jurisdictions compete to attract shares of the R&D investment budget of a large multinational enterprise, whose investments potentially confer positive spillovers on national firms. The competition among the countries to tax the firm’s rents is modelled as common agency. An advantage of the common agency approach is that it enables the tax systems to be endogenously determined, based on informational considerations. (In contrast, the tax competition literature typically imposes exogenous constraints on the available tax instruments.) The firm contributes to local welfare by these spillovers, by tax payments and by dividends paid to local investors. The firm has private information both about its efficiency and about spillovers. It is shown that strategic tax competition may lead to overinvestments relative to the first-best allocation, that the excessive investments occur in the country where the positive spillover effects are lowest, and that they are most severe for the least efficient firms. This occurs for sufficiently asymmetric spillovers, and implies that investments under competition are then excessively spread out and not properly concentrated to the country where spillovers would be largest.

The tax literature normally assumes that any one firm is too small to affect tax policy in a jurisdiction. We assume that the MNE is a large and unique firm with a high level of R&D, or that the jurisdictions are small, so that the potential tax revenues and the possible knowledge spillovers from the firm are non-negligible relative to the corporate tax bases and the knowledge bases of the two jurisdictions. An alternative interpretation is that the tax subject in the model is a mobile industry.

We assumed that the firm has private information about its efficiency, whereas its investment levels have been assumed to be subject to symmetric information. Profits may to a large extent may be observable for purely domestic firms, and be captured by a traditional corporate income tax. For multinational firms, transfer pricing may make any attempt to measure profits difficult, so that countries are forced to estimate profits based

\(^{29}\) See Markusen (1995).

\(^{30}\) See, e.g., Devereux and Freeman (1995).
on what is observable. Our assumption is that investments are the key such observable variable, and the tax schemes derived are made contingent on the national investment levels.\footnote{Privately observed investments that are undertaken after the tax system is in place (moral hazard) can be accommodated in the model; the profit function can be interpreted as an indirect function where such investments are chosen optimally, conditional on the observable $K_j$'s. Privately observed investments in place ex ante would, however, be a part of the firm's private information. The model represents a case where the aggregate effect of several such variables can be captured by a one-dimensional parameter.}

We assume that the MNE’s efficiency levels are perfectly correlated in the two countries of operation. Uncorrelated efficiency parameters may be relevant if firms invest in different countries to diversify portfolios. Asymmetric information about investment levels, or uncorrelated information parameters, may represent interesting extensions of the present model. However, each of these extensions would imply a multidimensional screening problem (i.e. a challenge for the government to reveal a vector of parameters subject to private information), which is not yet fully solved, not even in a single-principal setting; see Rochet and Chone (1998).

Appendix

R&D investments as substitutes.

Suppose the investment variables $K_i$ represent investments in product or process innovations, and that only the best of these innovations will be used by the firm. Assume that investments are risky, and let the stand-alone gross value of an innovation by affiliate $i$ be given by $a_iK_i + \varepsilon_i$, where $\varepsilon_i$ is a zero-mean random variable that captures the risk, and $a_i$ is a constant (but may depend on $\theta$). Investments are thus measured so that the expected stand-alone value is proportional to the investment made. The gross value (before costs) is then

\[
\Gamma(K_1, K_2) = E \max\{a_1K_1 + \varepsilon_1, a_2K_2 + \varepsilon_2\} = a_1K_1 + E \max\{\varepsilon_1 - \varepsilon_2, a_2K_2 - a_1K_1\}
\]

This can be written as

\[
\Gamma(K_1, K_2) = a_1K_1 + \phi(a_2K_2 - a_1K_1)
\]

where

\[
\phi(x) = E \max\{\varepsilon, x\} = \bar{\varepsilon} - \int_{x}^{\bar{\varepsilon}} F(\varepsilon)d\varepsilon
\]

and $\varepsilon = \varepsilon_1 - \varepsilon_2$, $F(\varepsilon)$ is the CDF for this difference, and $\bar{\varepsilon}$ is the upper bound for its realizations. Note that $\phi'(x) = F(x)$, and hence that investments are substitutes with
respect to gross value; $\frac{\partial^2 \Pi}{\partial K_2 \partial K_1} < 0$. They are then also substitutes with respect to profits $\Pi = \Gamma - C$ as long as investment costs satisfy $\frac{\partial^2 C}{\partial K_2 \partial K_1} \geq 0$. Efficiency $\theta$ may enter via the parameters $a_i$ or via the distribution $F()$.

In the case where $\varepsilon = \varepsilon_1 - \varepsilon_2$ is uniform, we find that $\phi(x)$ is quadratic; $\phi(x) = x + \frac{1}{2} (\varepsilon - x)^2$, and hence that the gross value is quadratic also. The quadratic specification in Section 5 can thus represent such a case.

**Derivation of non-cooperative equilibrium conditions.**

Consider the optimisation problem for country 1. Integrating by parts, and using (5), the expected welfare in country 1 may be written

$$EW = \int_{\theta} \{ (1 + \lambda) (\Pi(K_1(\theta), K_2(\theta), \theta) + E_1(K_1(\theta), \theta)) - R_2(K_2(\theta)) \} \ dF(\theta).$$

Maximising the integrand pointwise with respect to $K_1$ and $K_2$, subject to (7), we obtain the first-order conditions

$$0 = (1 + \lambda) (\frac{\partial \Pi(K_1, K_2, \theta)}{\partial K_1} + \frac{\partial E_1(K_1, \theta)}{\partial K_1}) f(\theta)$$

$$(1 + \lambda - \alpha_1) \frac{\partial^2 \Pi(K_1, K_2, \theta)}{\partial \theta \partial K_1} (1 - F(\theta)) - \mu(\theta) \frac{\partial^2 \Pi(K_1, K_2, \theta)}{\partial K_1 \partial K_2},$$

and

$$-(1 + \lambda - \alpha_1) \frac{\partial^2 \Pi(K_1, K_2, \theta)}{\partial \theta \partial K_2} (1 - F(\theta))$$

$$+ \mu(\theta) \left[ \frac{\partial^2 \Pi(K_1, K_2, \theta)}{\partial K_2^2} + \frac{\partial^2 \Pi(K_1, K_2, \theta)}{\partial K_1 \partial K_2} - R_2'(K_2) \right] = 0,$$

where $\mu(\theta)$ is a multiplier corresponding to the constraint (7). Differentiating Eq.(7) with respect to $\theta$ we get

$$\frac{\partial^2 \Pi(K_1, K_2, \theta)}{\partial K_2^2} K_2'(\theta) + \frac{\partial^2 \Pi(K_1, K_2, \theta)}{\partial \theta \partial K_2} + \frac{\partial^2 \Pi(K_1, K_2, \theta)}{\partial K_1 \partial K_2} K_1'(\theta) - R_2'(K_2) K_2'(\theta) = 0.$$

Combining (16) and (17), we obtain $\mu(\theta) = - \frac{(1 + \lambda - \alpha_1) \frac{\partial^2 \Pi}{\partial K_2 \partial K_1} (1 - F)}{\frac{\partial^2 \Pi}{\partial K_2^2} + \frac{\partial^2 \Pi}{\partial K_1 \partial K_2} K_1'(\theta)}$, and by inserting for $\mu(\theta)$ in (15) we obtain the condition (8) (with $j = 1$ and $i = 2$) that characterises the equilibrium contract for country 1.
It should also be checked that the solution is commonly implementable. Given the associated tax functions, it must indeed be optimal for the agent to report truthfully to both principals (i.e. \( \pi(\theta, \theta) \geq \pi(\hat{\theta}_1, \theta) \) for all feasible reports \( \hat{\theta}_1 \) in principal 1’s problem, and similarly for principal 2), and make the targeted investments \( K_1(\theta), K_2(\theta) \). The latter requirement is fulfilled if \( \Pi(K_1, K_2, \theta) - \Sigma_j R_j(K_j) \) is concave.\(^{32}\) Local concavity is necessary at the point \( (K_1, K_2) = (K_1(\theta), K_2(\theta)) \). At that point we have from (17)\(^{\text{\textsuperscript{Proof of formula (13).}}\text{\textsuperscript{32}}} \)

\[
\frac{\partial^2 \Pi}{\partial K_j^2} + \frac{\partial^2 \Pi}{\partial K_1 \partial K_2} - R'_j = -\frac{i}{K_j}(\frac{\partial^2 \Pi}{\partial K_1^2} + \frac{\partial^2 \Pi}{\partial K_1 \partial K_2} K'_j),
\]

and the necessary local concavity conditions can then be written as \( K'_1 K_2' + K_1' K_2 + \frac{\partial^2 \Pi}{\partial K_1 \partial K_2} K'_i \geq 0 \), \( i = 1, 2 \), and

\[
K'_1 K'_2 \left( \frac{\partial^2 \Pi}{\partial K_1^2} + \frac{\partial^2 \Pi}{\partial K_1 \partial K_2} K'_1 + \frac{\partial^2 \Pi}{\partial K_2^2} K'_2 \right) \geq 0.
\]

These conditions are also sufficient in the case of quadratic functions and contract substitutes, provided both investment schedules are nondecreasing (cf. Stole (1992), Thm. 11, p. 22). For the parametrisations given in Section 5, where \( \frac{\partial \Pi}{\partial K_1} = m \) and \( \frac{\partial \Pi}{\partial K_1} = -a \), the conditions then amount to

\[-a K'_1 K'_2 + m K'_1 \geq 0, \quad i = 1, 2 \text{ and } K'_1 K'_2 (m^2 - a [m K'_1 + m K'_2]) \geq 0.\]

**Proof of formula (13).**

From \( K_j(\theta) = \hat{K}_j - K'_j \cdot (1 - \theta) \) we obtain the inverse \( \theta_j(K_j) = 1 - \frac{\hat{K}_j - K'_j}{K'_j} \) and hence (explanations to follow)

\[
R'_j(K_j) = m(\theta_j(K_j) - 1) + m + k - (q + a)K_j - aK_i(\theta_j(K_j)) \quad \text{(10)}
\]

\[= -m \frac{\hat{K}_j - K'_j}{K'_j} + (q + a) \hat{K}_j - K'_j + a \hat{K}_i - e_j - a \left( \hat{K}_i - K'_i \frac{\hat{K}_j - K'_j}{K'_j} \right) \]

\[= \left( -m \frac{\hat{K}_j - K'_j}{K'_j} + (q + a) \right) \left( \hat{K}_j - K'_j \right) - e_j \]

The first equality follows from (10) and the expression for \( \frac{\partial \Pi}{\partial K_j} \), the second by inserting for \( \theta_j(K_j) \) and for \( K_j(\theta) \), and by taking account of the fact that the first-best investments \( \hat{K}_j, \hat{K}_i \) satisfy \( \frac{\partial \Pi}{\partial K_j}(\hat{K}_j, \hat{K}_i, \theta) = e_j + m + k - (q + a) \hat{K}_j - a \hat{K}_i = 0 \). This proves the formula. Note that the expression in the first parenthesis in the last line has the same sign as \(-(m - a K'_j - (q + a) K'_j))\), and that this is positive according to (11). Hence we see that \( R'_j < 0 \).

**Proof of Proposition 3.**

Comparing the slopes of the investment schedules for the cooperative and the non-

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\(^{32}\) Concavity holds if \( \frac{\partial^2 \Pi}{\partial K_j^2} + \frac{\partial^2 \Pi}{\partial K_1 \partial K_2} - R'_j \leq 0 \) and \( \left( \frac{\partial^2 \Pi}{\partial K_1^2} + \frac{\partial^2 \Pi}{\partial K_1 \partial K_2} - R'_1 \right) \left( \frac{\partial^2 \Pi}{\partial K_2^2} + \frac{\partial^2 \Pi}{\partial K_1 \partial K_2} - R'_2 \right) - \left( \frac{\partial^2 \Pi}{\partial K_1 \partial K_2} \right)^2 \geq 0.\)
cooperative solutions we have
\[
1 - \frac{K'_{jC}}{K'_j} = 1 - \frac{1+\varepsilon + \gamma}{Q+1} x = -\varepsilon + Q + 2 (\gamma_1 - \gamma) - \sqrt{\varepsilon^2 - 2Q\varepsilon + 4\varepsilon \gamma_1 + 4\gamma_1 + Q^2 + 4\gamma_1^2}{2(Q+1)x},
\]

where \(\gamma = 1 - \frac{2a_i}{1+\lambda}\) and \(\gamma_1 = 1 - \frac{a_i}{1+\lambda}\), so \(\gamma_1 - \gamma = \frac{a_i}{1+\lambda}\). There is underinvestment in common agency relative to the cooperative case if \(K'_j > K'_jC\), i.e. \(-\varepsilon + Q + 2 (\gamma_1 - \gamma) > 0\) and \((-\varepsilon + Q + 2 (\gamma_1 - \gamma))^2 > (\varepsilon^2 - 2Q\varepsilon + 4\varepsilon \gamma_1 + 4\gamma_1 + Q^2 + 4\gamma_1^2)\). The latter inequality means \(-\varepsilon (2\gamma_1 - \gamma) + Q (\gamma_1 - \gamma) + (\gamma (2\gamma_1 - \gamma) - \gamma > 0\). Noting that \(2\gamma_1 - \gamma = 1\), the last inequality is equivalent to \(-\varepsilon + Q (\gamma_1 - \gamma) - \gamma > 0\). This yields \(K'_j > K'_jC\) if \(\frac{q}{a} + 4 > \frac{1+\lambda}{a_1}(\frac{\varepsilon}{\gamma} + 2)\). QED

**Proof of Lemma.**

Define ”normalized” slope parameters \(x_j = \frac{a_i}{m}K'_j\). In terms of these parameters, equations (11) take the form

\[
1 + \varepsilon_1 - Qx_1 - x_2 = -\gamma_1 \left[1 + \frac{x_2}{x_1 - 1}\right], \quad 1 - Qx_2 - x_1 = -\gamma_1 \left[1 + \frac{x_1}{x_2 - 1}\right]
\]

From the second of these equations it follows that \(x_1 = \frac{1-Qx_2+\gamma_1}{1-x_2+\gamma_1}(1-x_2)\) (provided \(1 - x_2 + \gamma_1 \neq 0\)). Substituting this into the first, we find that \(x_2\) is a root of a third-order equation \(Ax_2^3 + Bx_2^2 + Cx_2 + D = P(x_2) = 0\), where

\[
A = Q^3 - Q, \quad B = -2Q^2\gamma_1 - Q^2 + \varepsilon_1 Q + 3Q - 2Q^3 + 2Q\gamma_1, \quad C = 3Q^2\gamma_1 - 3Q\gamma_1 + Q^3 - 3Q - 2\varepsilon_1 Q - \varepsilon_1\gamma_1 + 2Q^2 - \varepsilon_1\gamma_1 Q, \quad D = (\gamma_1 + 1) (\varepsilon_1\gamma_1 + Q - Q^2 + \varepsilon_1 Q)
\]

We look for a solution pair that satisfies the implementability conditions \(0 \leq x_j \leq 1\) and \(x_1 + x_2 \leq 1\). The polynomial satisfies \(P(0) = D\) and \(P(1) = \varepsilon_1\gamma_1^2 > 0\). It has a root in \([0, 1]\) if \(D \leq 0\), and there is then only one root in this interval. The root is positive when \(D < 0\). Note that \(D \leq 0\) for \(\varepsilon_1 \leq Q\frac{Q-1}{\gamma_1+Q} = \varepsilon_1^0\). For \(0 < \varepsilon_1 \leq \varepsilon_1^0\) there is thus a unique root satisfying \(0 \leq x_2 < 1\), and the root is \(x_2 = 0\) if \(\varepsilon_1 = \varepsilon_1^0\).

It follows that \(x_1 = \frac{1-Qx_2+\gamma_1}{1-x_2+\gamma_1}(1-x_2)\) satisfies \(0 < x_1 \leq 1\), with \(x_1 = 1\) if \(x_2 = 0\). Note also that the formula for \(x_1\) yields \(x_1 + x_2 - 1 = \frac{(1-Q)x_2}{1-x_2+\gamma_1}(1-x_2)\). Thus we have \(x_1 + x_2 \leq 1\) with equality if \(x_2 = 0\), i.e. \(\varepsilon_1 = \varepsilon_1^0\). These results prove the lemma.

**Proof of Proposition 5.**
The cooperative investment schedules have slopes \( K'_{JC} = K'_{jF} + \gamma \frac{m}{a} \frac{1}{Q-1}, \) where \( \gamma = 1 - \frac{2a}{1+\lambda}, \) and \( K'_{jF} \) is the slope of the corresponding first-best schedule. Setting \( \epsilon_2 = 0 \) and letting \( \epsilon_1 \to m\varepsilon_1^0 \) we find \( K'_{1F} \to \frac{m}{a} \frac{(1+\varepsilon_1^0)Q-1}{Q^2-1}, \) and hence

\[
K'_{1C} \to \frac{m}{a} \frac{(1+\varepsilon_1^0)Q-1}{Q^2-1} + \gamma \frac{m}{a} \frac{1}{Q-1} = \frac{m}{a(Q+1)} \left[ \frac{Q^2}{Q+\gamma_1} + 2\gamma_1 \right]
\]

where the last equality follows from the definition of \( \varepsilon_1^0 \) and the fact that \( \gamma = 2\gamma_1 - 1. \) From the lemma above we know that the non-cooperative solution satisfies \( K'_1 \to \frac{m}{a} \) in this case.

In the limit we thus have non-cooperative investments being lower than cooperative ones in country 1 iff \( \frac{m}{a} > \frac{m}{a(Q+1)} \left[ \frac{Q^2}{Q+\gamma_1} + 2\gamma_1 \right]. \) This condition is equivalent to the condition given in Proposition 5. QED.

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Figure 1. First-best (heavy lines) and second-best cooperative (thin lines) investments.

Figure 2. First-best (heavy lines) and common agency (thin lines) investments